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Lecture 09 – 2D Features

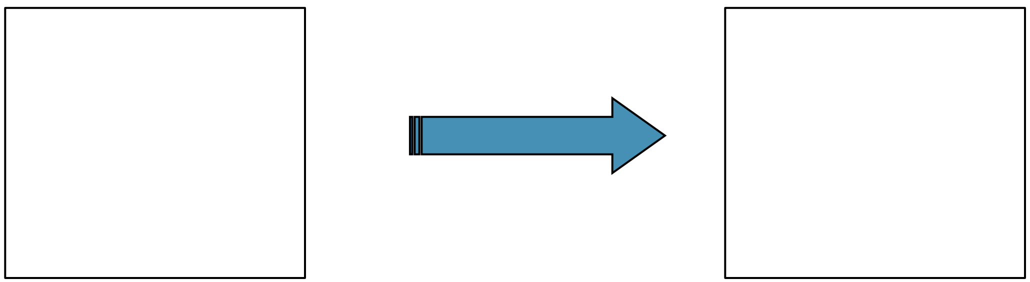
Digital Image Processing

- Digitization (sampling + quantizing)
- Preprocessing (contrast and brightness changes, denoising, sharpening, …)
- Image analysis (object detection and recognition, scene understanding)
- Image coding (compression)

Topics of ROZ2

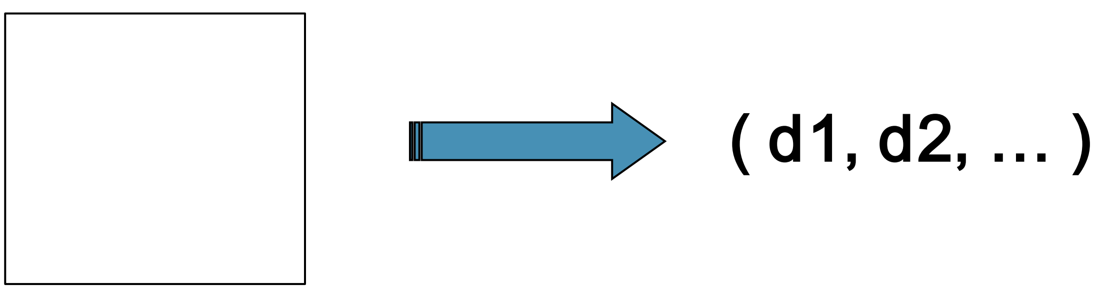
- Digitization (sampling + quantizing)
- Preprocessing (contrast and brightness changes, denoising, sharpening, …)
- Image analysis (object detection and recognition, scene understanding)
- Image coding (compression)

Image (pre)processing



Image

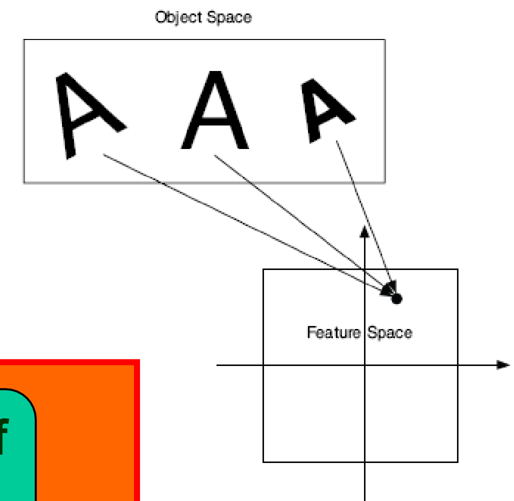
Image analysis

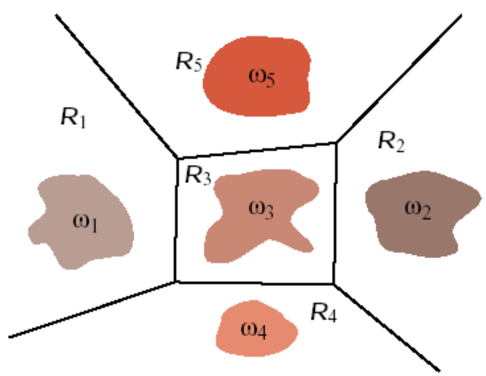


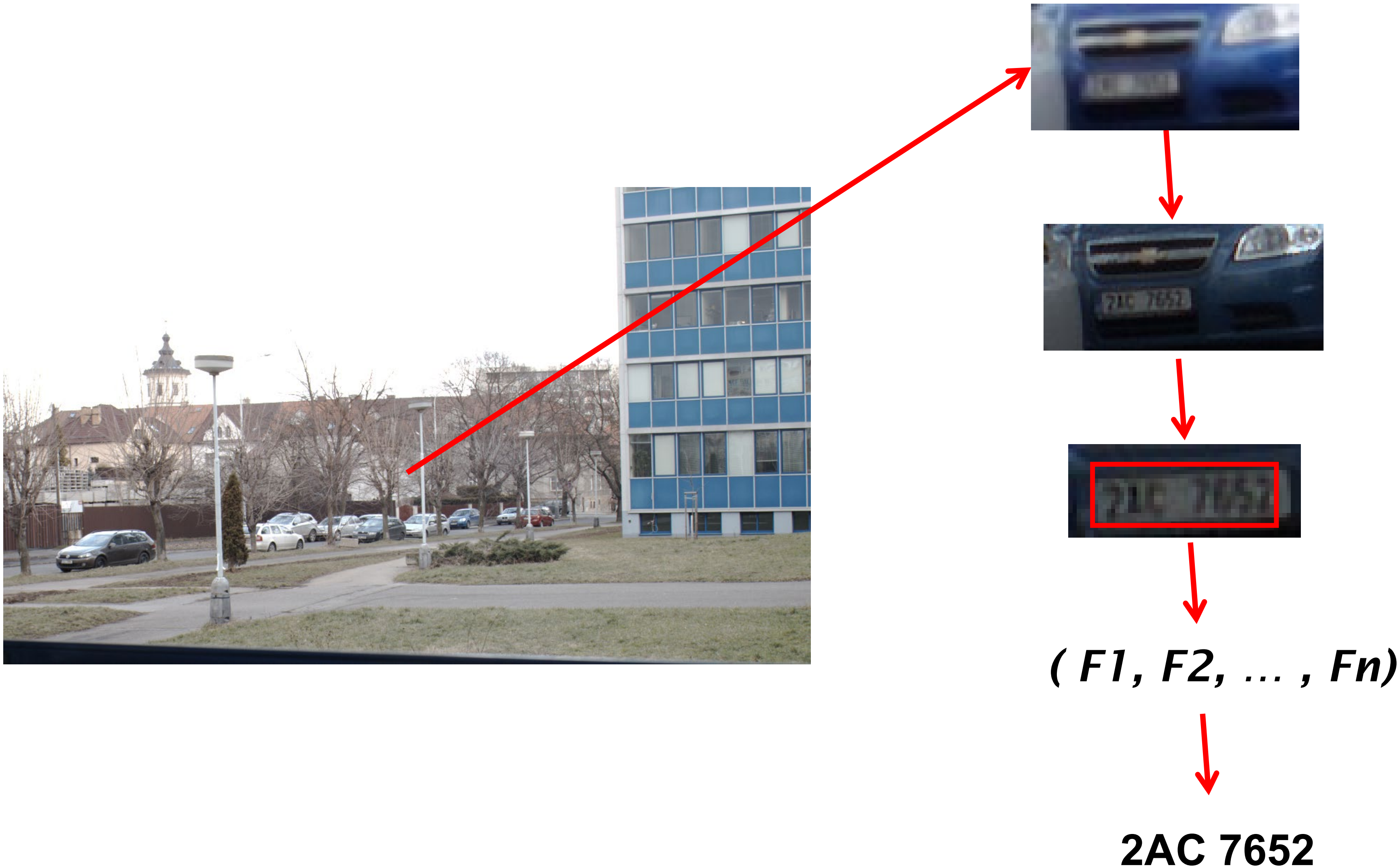
Non-image

Object Recognition System

- Image acquisition
- Preprocessing
- Object detection
- Computing of the features
- Classification
- Class label






(F1, F2, … , Fn)
2AC 7652

Why is visual object recognition so difficult for machines?

Human beings

- use their lifetime experience as a prior

Why is visual object recognition so difficult for machines?

Human beings

- use supplementary information (sound, touch)

Why is visual object recognition so difficult for machines?

Human beings

- are very robust to object degradations

Why is visual object recognition so difficult for machines?

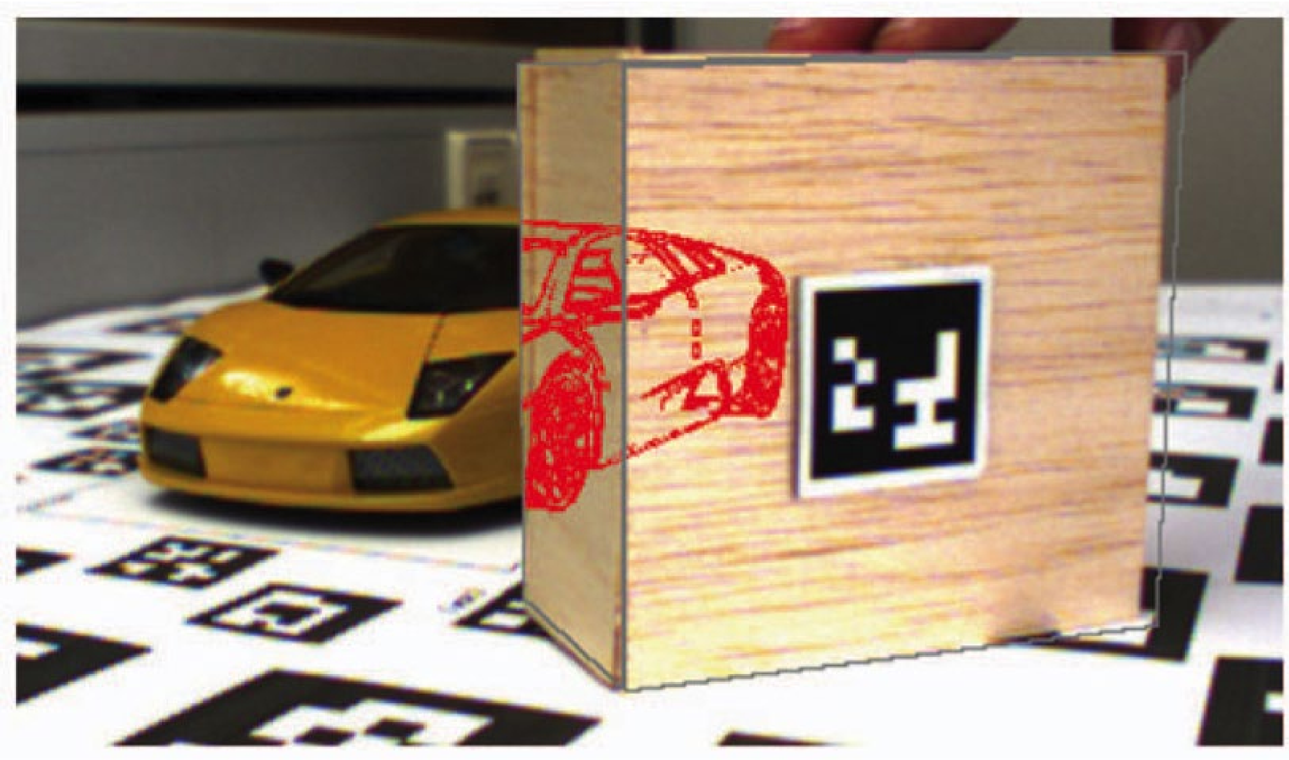
Human beings

- learn not only from the previous results but also from the recognition process itself

Why is visual object recognition so difficult for machines?

Human beings

- can efficiently work with incomplete information



Why is visual object recognition so difficult for machines?

Human beings

- can use a broad context

Features for description and recognition of 2D objects

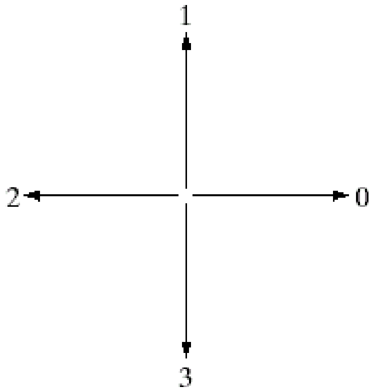
Feature = a point in a metric space (usually in *n*-D Euclidean space) that describes the object

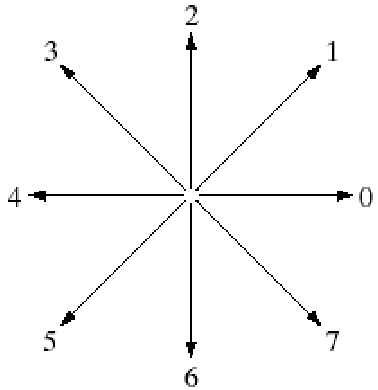
What is a 2-D object?

- Binary
- Finite
- Boundary – a simple closed curve or a finite set of them

What is a (discrete) boundary?

- Boundary pixel – an object pixel having a background neighbor
- What is a neighbor? (Definition of discrete topology)

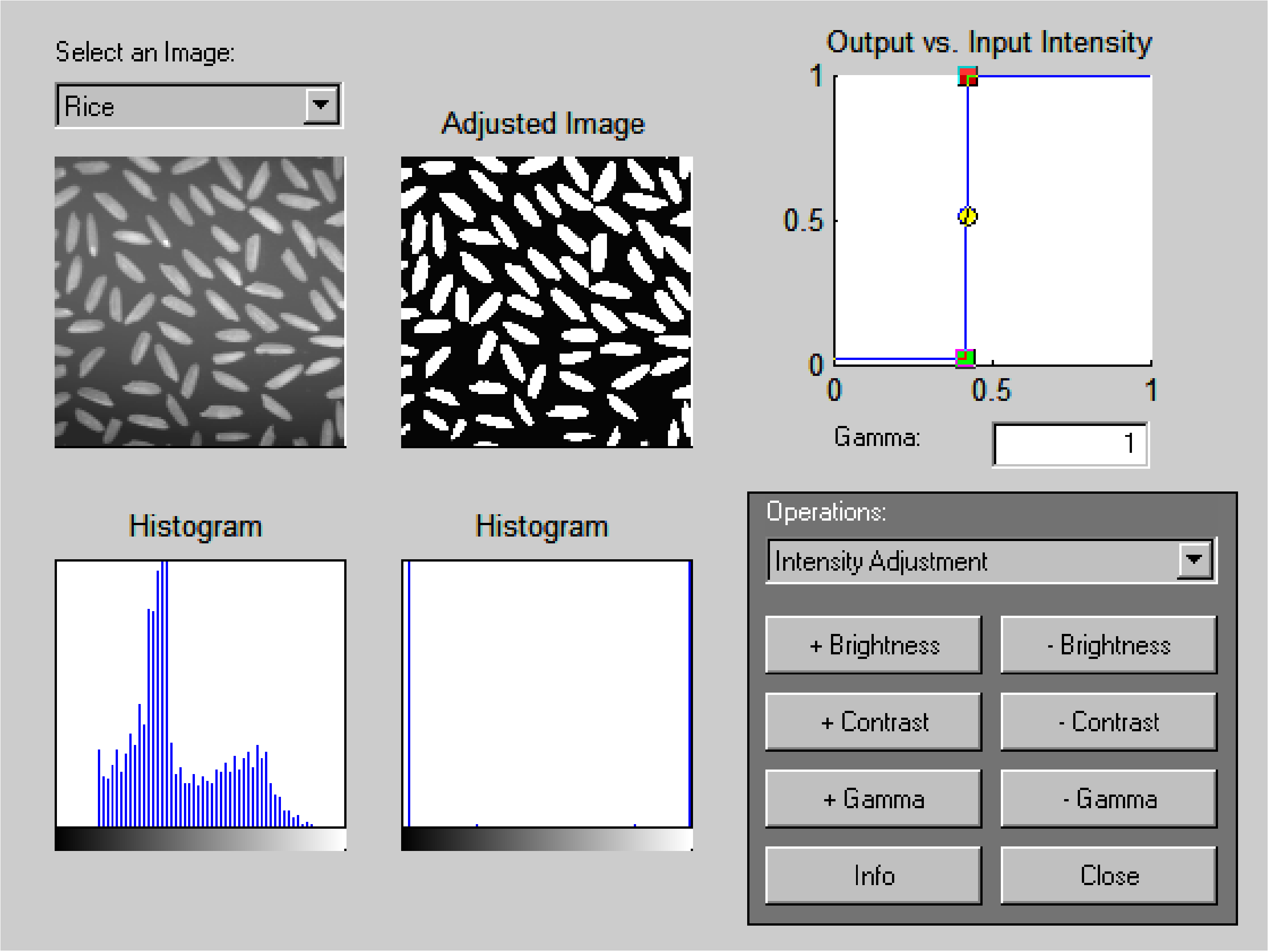




How to detect objects in the image? (Image segmentation)

- Thresholding
- Edge linking
- Region growing

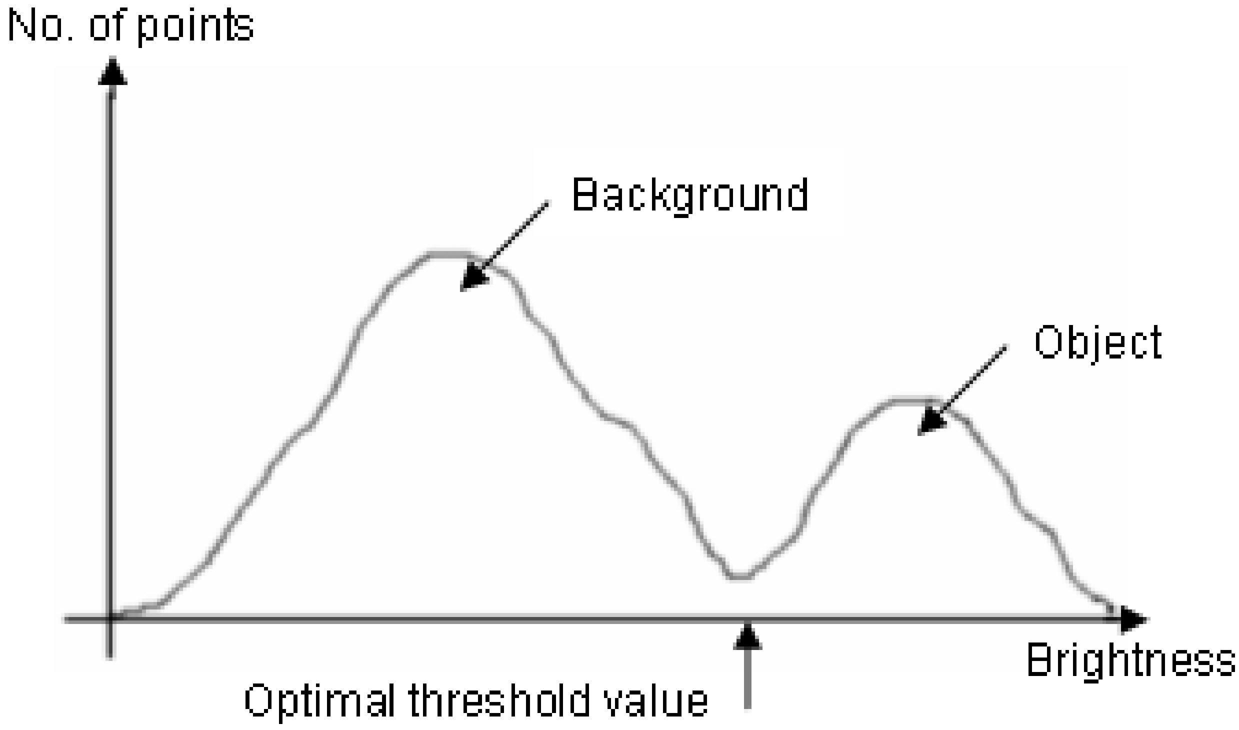
Thresholding



The Otsu's threshold

$$\sigma_w^2(t) = \omega_1(t)\sigma_1^2(t) + \omega_2(t)\sigma_2^2(t)$$

$$\sigma_b^2(t) = \sigma^2 - \sigma_w^2(t) = \omega_1(t)\omega_2(t)\left[\mu_1(t) - \mu_2(t)\right]^2$$



The Otsu's threshold

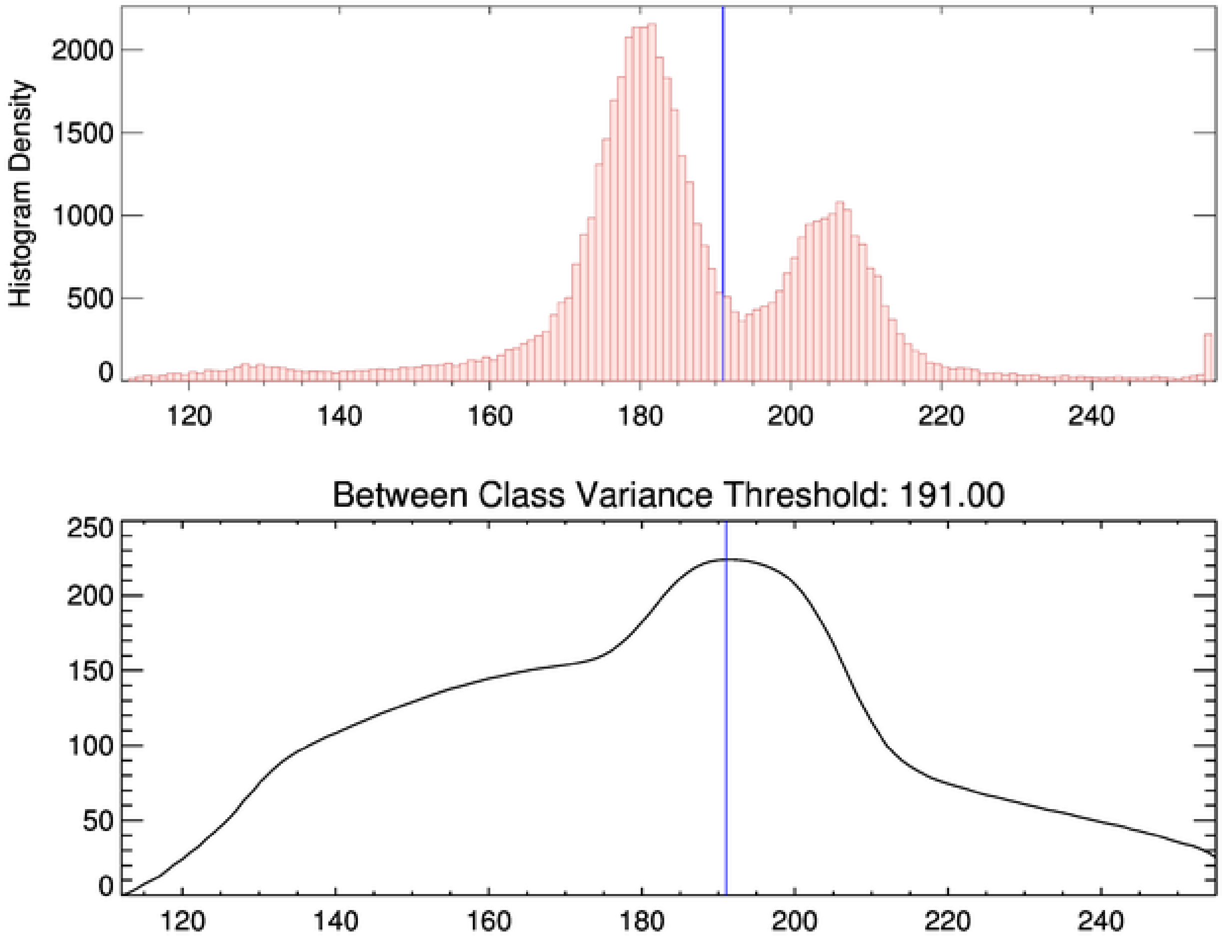
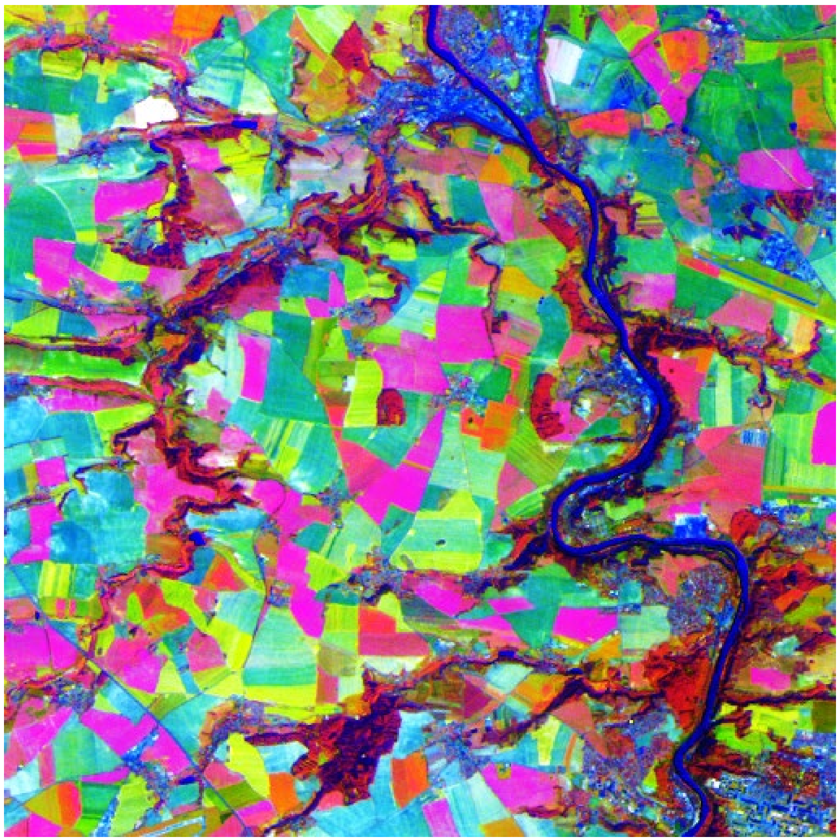
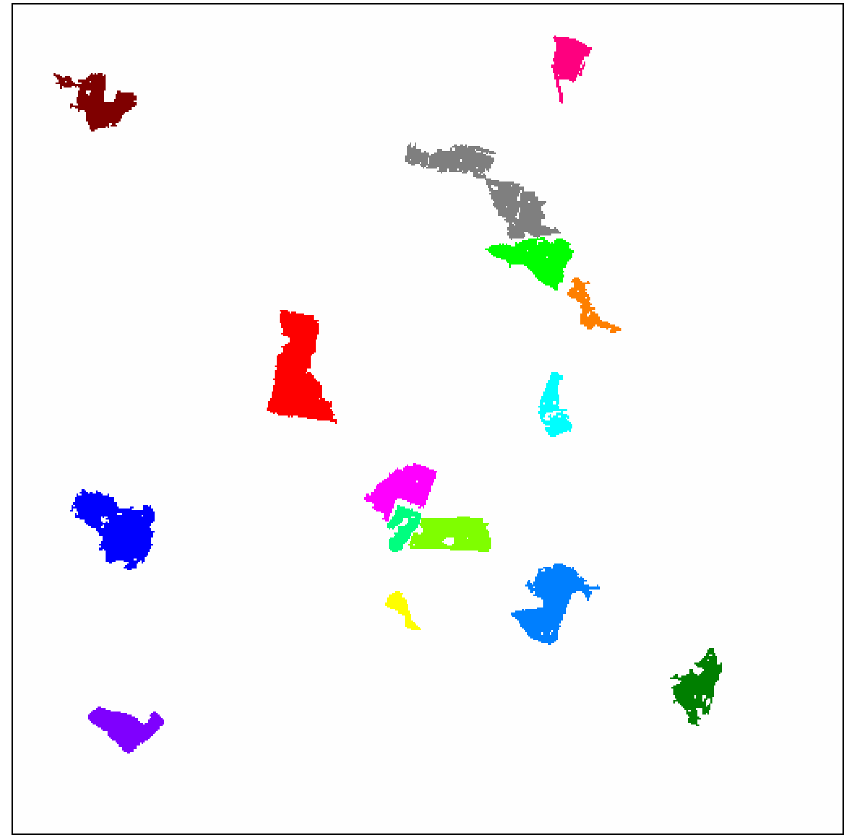


Image segmentation



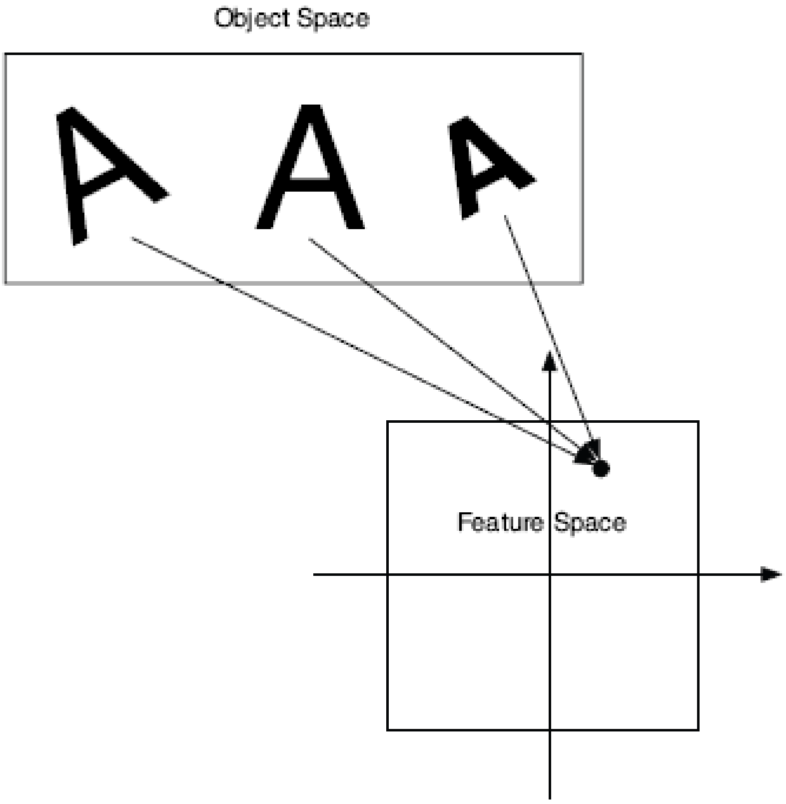
Original



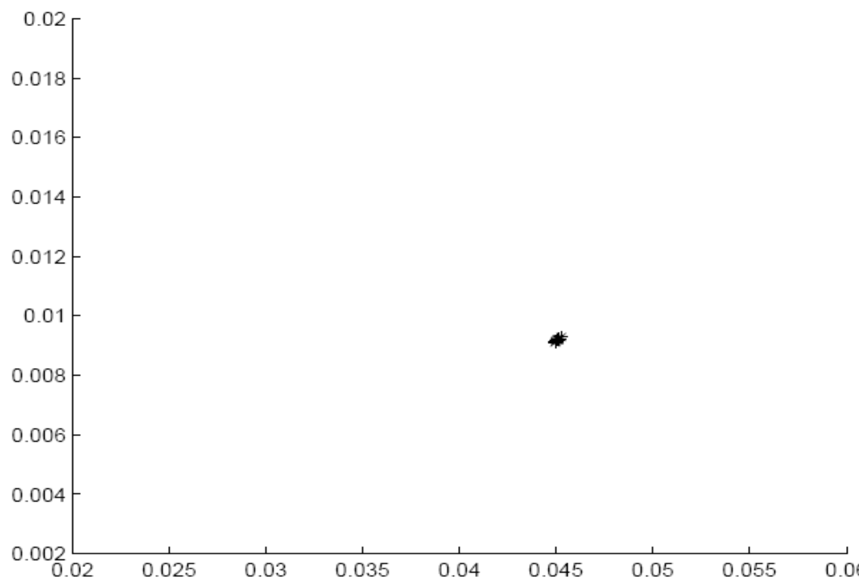
Partial segmentation

Full segmentation

What are features?



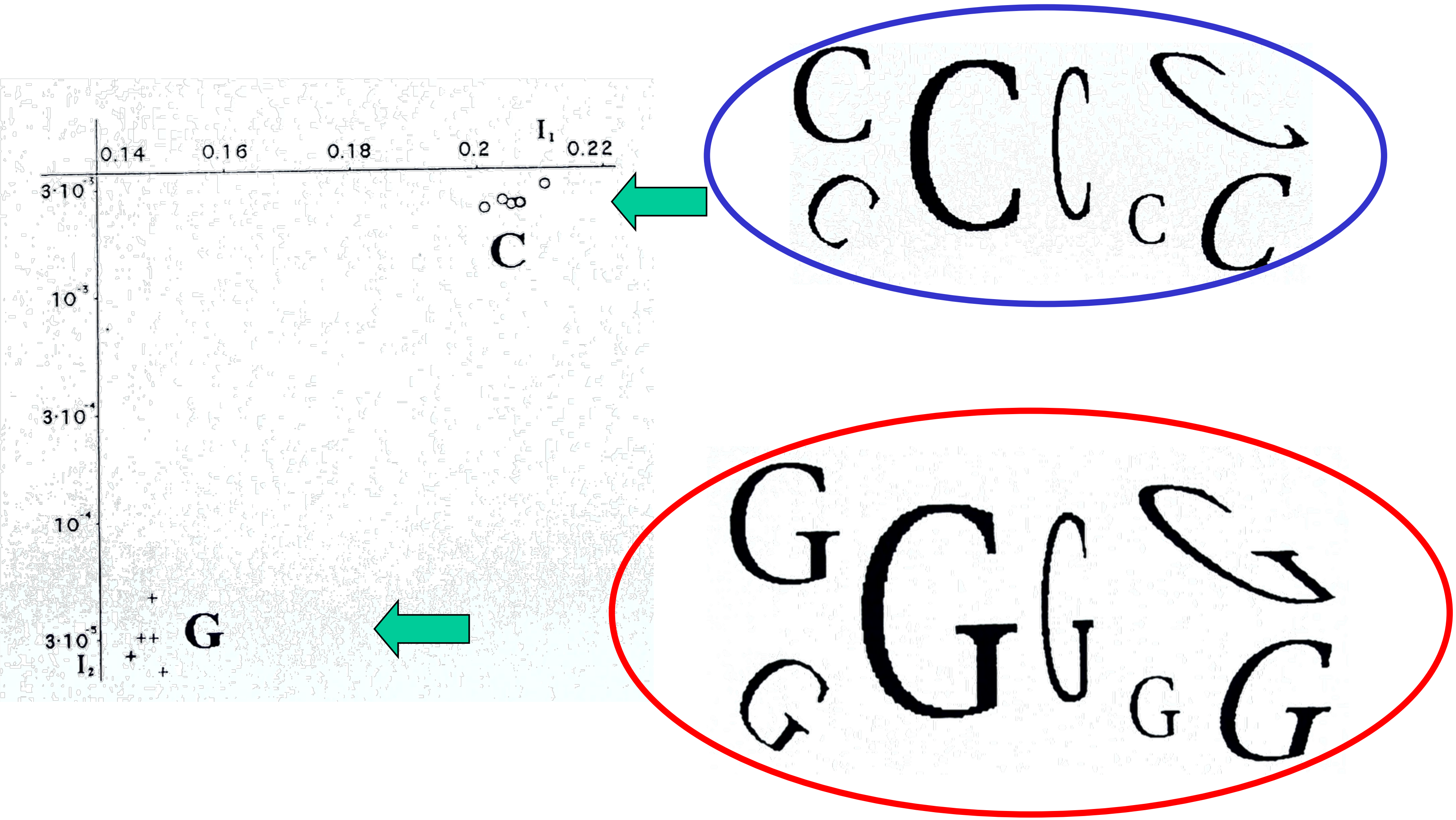
Example: TRS


0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0.02
0.025
0.03
0.035
0.04
0.045
0.05
0.055
0.06

Desirable properties of the features

- Invariance
- Discriminability
- Robustness
- Efficiency, independence, completeness

Discrimination power



Major categories of invariants

Simple “visual” shape descriptors

- compactness, convexity, elongation, ...

Transform coefficient invariants

- Fourier descriptors, wavelet features, ...

Point set invariants

- positions of dominant points

Differential invariants

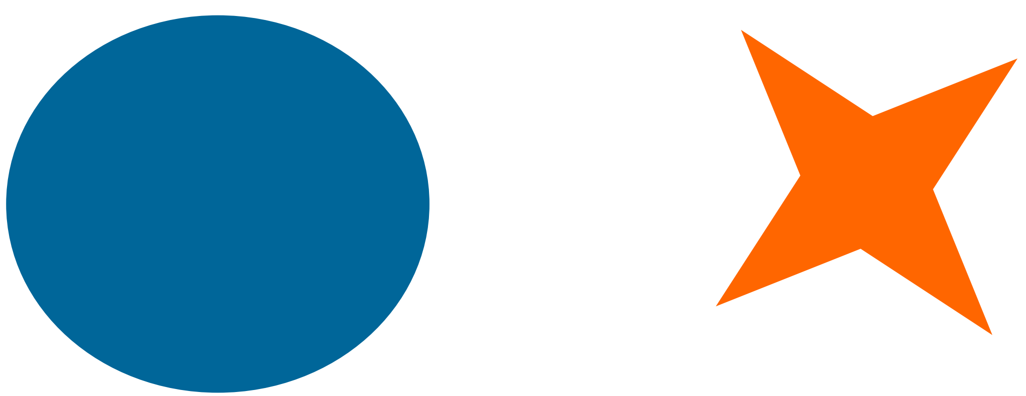
- derivatives of the boundary

Moment invariants

Visual features for binary objects

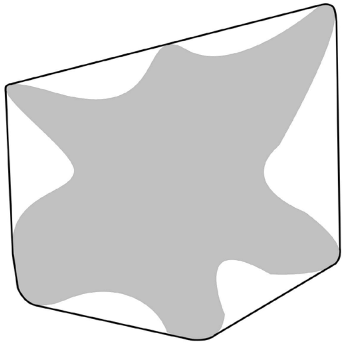
Simple features

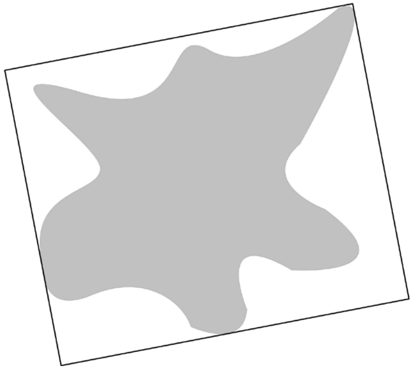
- Compactness $\frac{4\pi P}{O^2}$
- Convexity $\frac{P(A)}{P(C_A)}$



Visual features for binary objects

Simple features





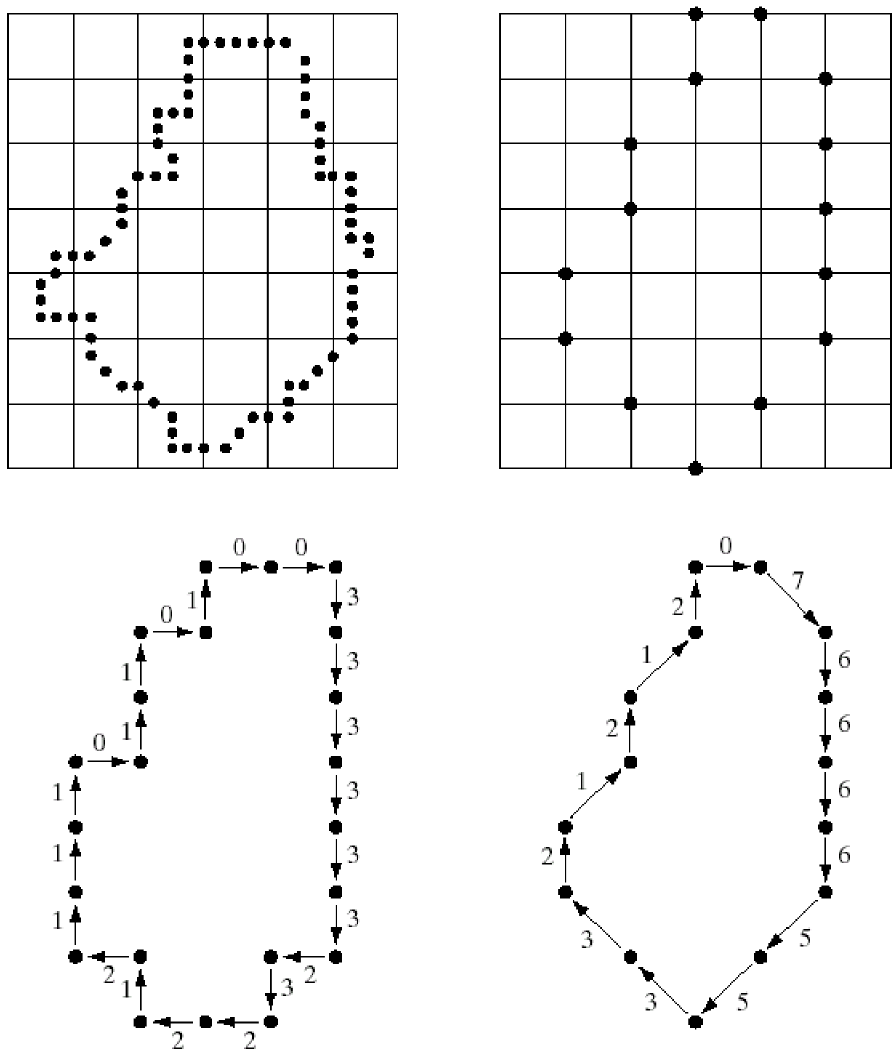
- Compactness $\frac{4\pi P}{O^2}$
- Convexity $\frac{P(A)}{P(C_A)}$
- Elongation
- Rectangularity
- Euler number

Visual features for binary objects

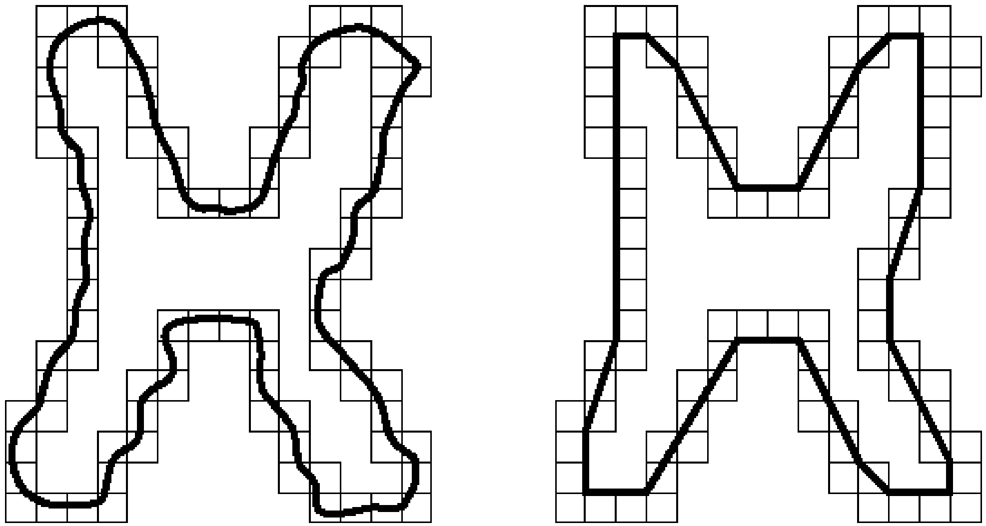
“Complete” features

- Chain code
- Polygonal approximation
- Shape vector
- Shape matrix
- Other encodings of the radial function

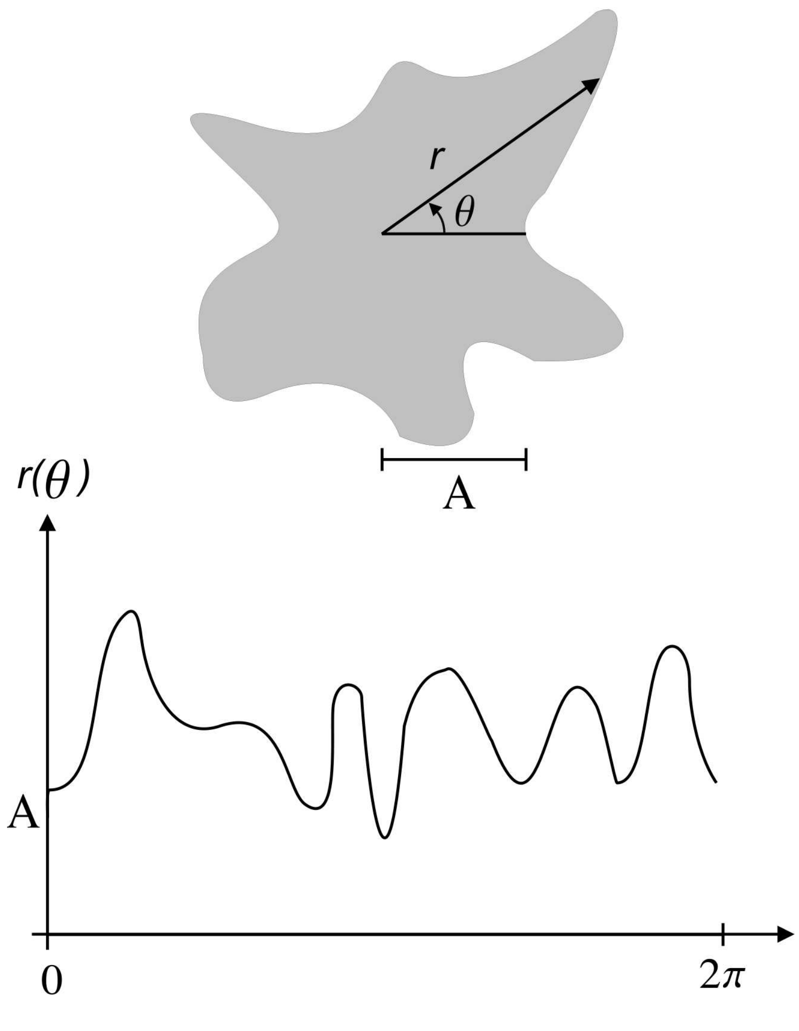
Chain code



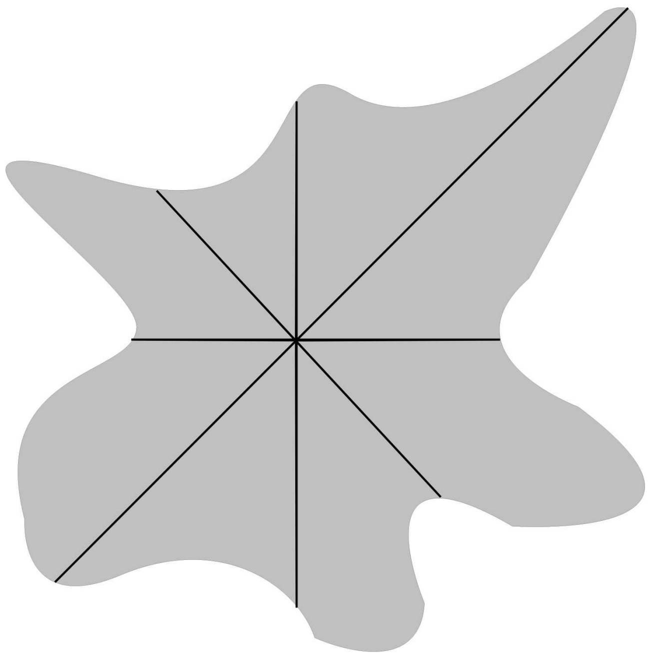
Polygonal approximation



Radial function



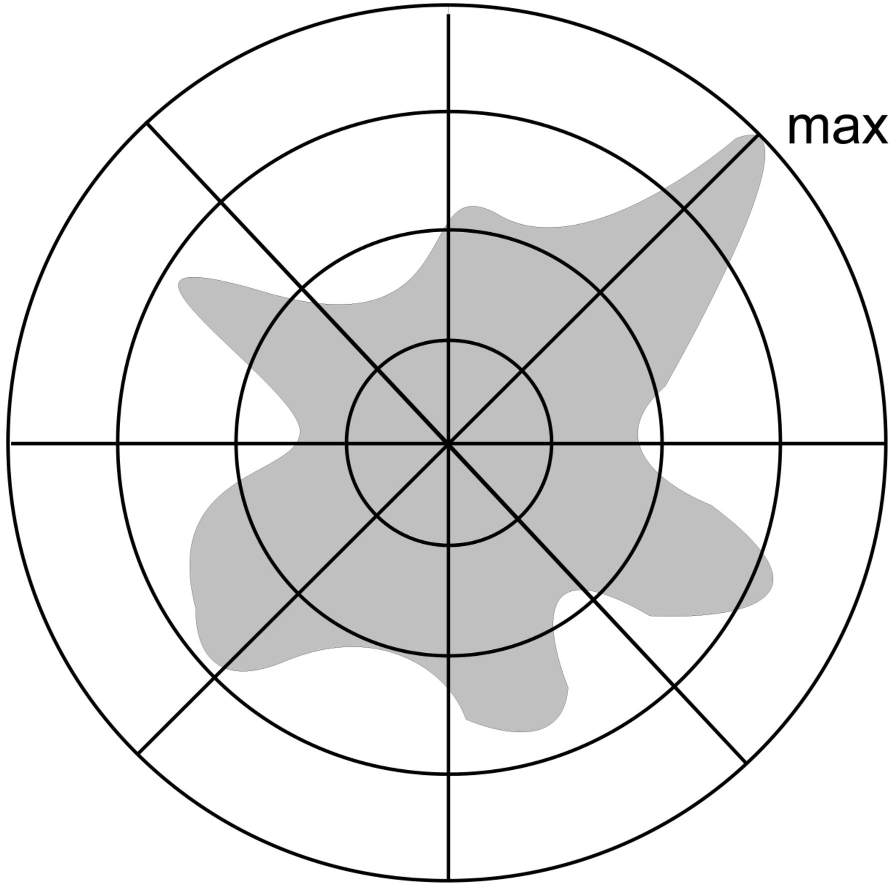
Shape vector



$$v = (d1, d2, \ldots, dn)$$

$n = 8$

Shape matrix



$m = 4$

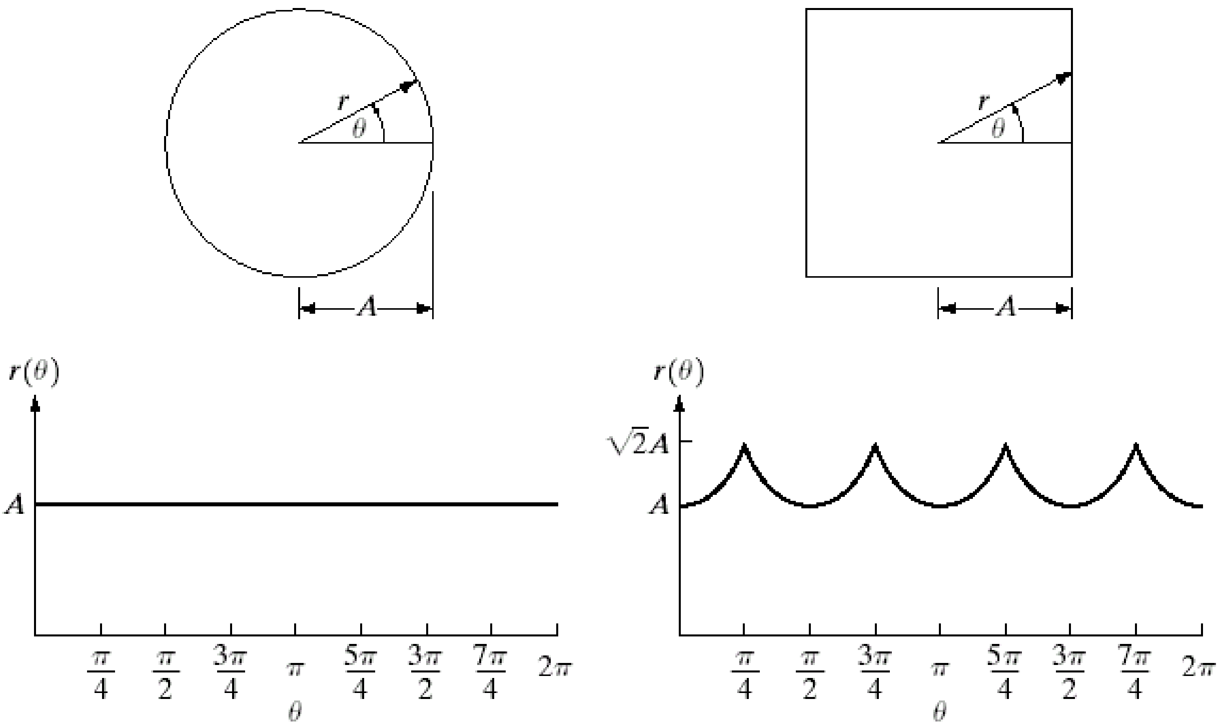
$n = 8$

$$B = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Transform coefficient features

- Fourier descriptors
- Wavelet-based features
- Other transform coefficients

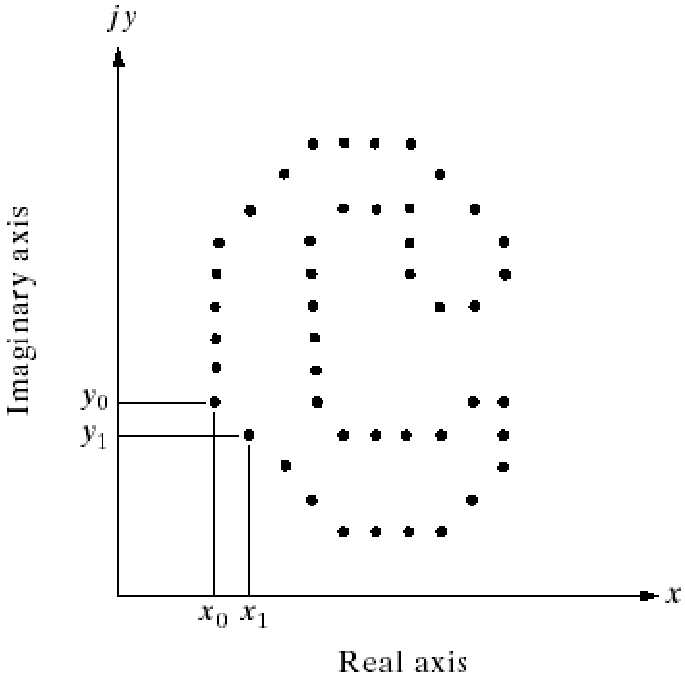
Fourier descriptors



$$f(t) = ([x(t) - x_c]^2 + [y(t) - y_c]^2)^{1/2}$$

Fourier descriptors

$$z_n = x_n + i \cdot y_n$$



$$Z_k = \sum_{n=0}^{N-1} z_n e^{-2\pi ikn/N}$$

$$C_k = |Z_k|/|Z_1|,\ k = 2, 3, \ldots$$

Shift invariance

$$\sum_{n=0}^{N-1} (z_n - z) e^{-2\pi i k n/N} = \sum_{n=0}^{N-1} z_n e^{-2\pi i k n/N} + z \sum_{n=0}^{N-1} e^{-2\pi i k n/N}$$

$$\sum_{n=0}^{N-1} e^{-2\pi i k n/N} = 0, \; k \neq 0$$

$$\sum_{n=0}^{N-1} e^{-2\pi i k n/N} = N, \quad k = 0$$

Rotation invariance

$$\sum_{n=0}^{N-1} (z_n e^{\varphi i}) e^{-2\pi i k n/N} = e^{\varphi i} \sum_{n=0}^{N-1} z_n e^{-2\pi i k n/N}$$

Scaling invariance

$$\sum_{n=0}^{N-1} (cz_n) e^{-2\pi i k n/N} = c \sum_{n=0}^{N-1} z_n e^{-2\pi i k n/N}$$

Invariance to the starting point

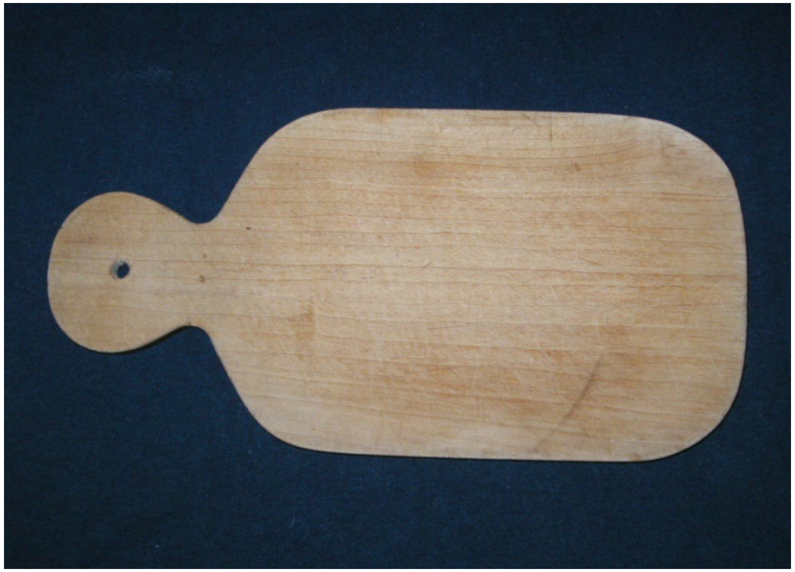
- Fourier Shift Theorem

Wavelet features

Differential invariants

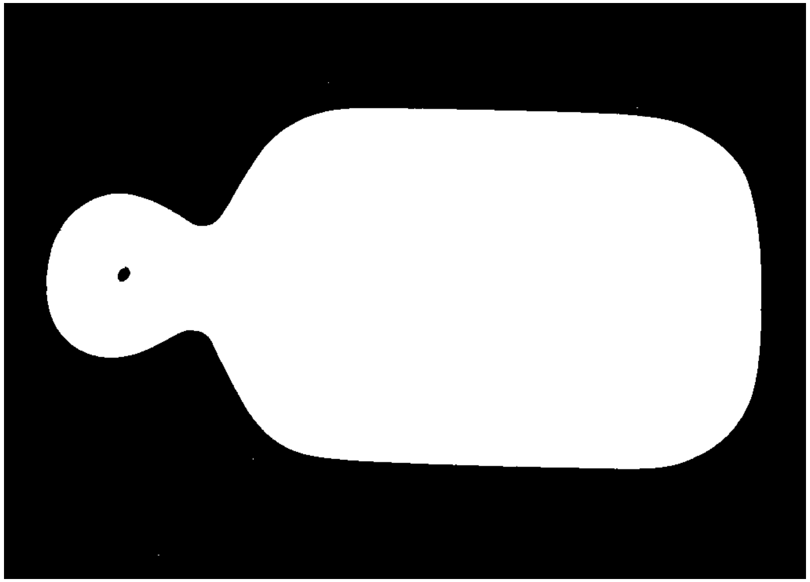
Motivation: recognition of occluded objects

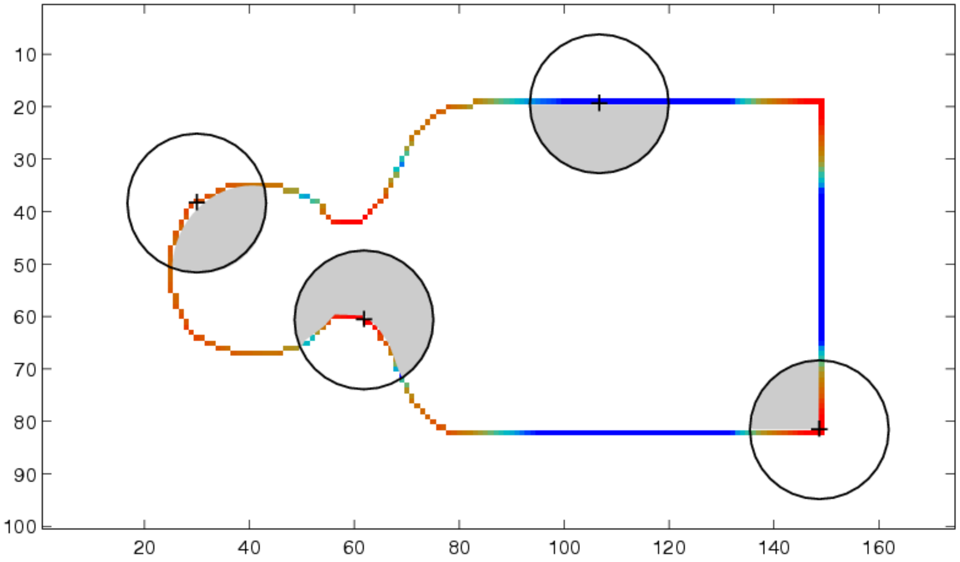
→ Features must be local



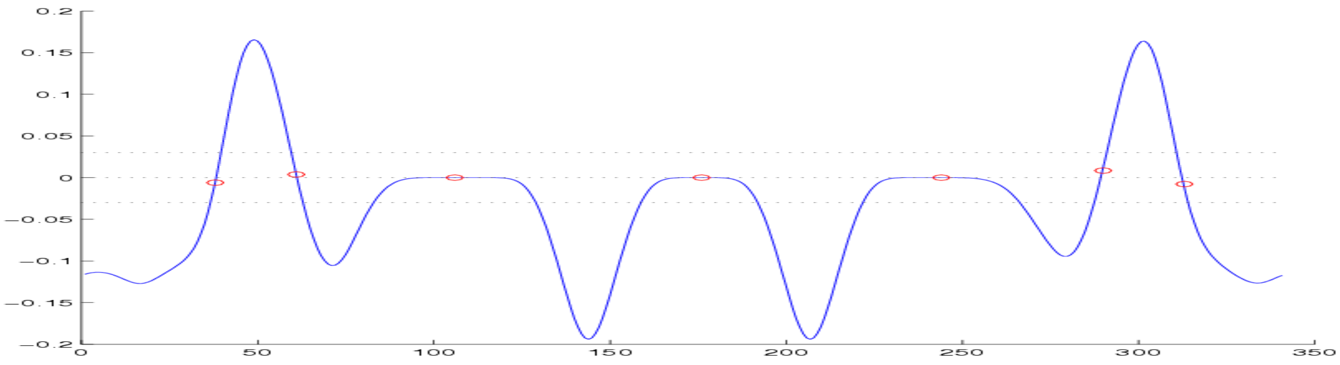
Differential invariants – an example

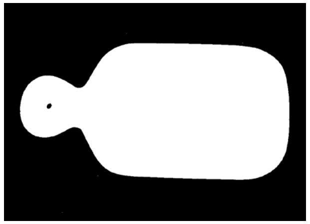
$$c(t) = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$$

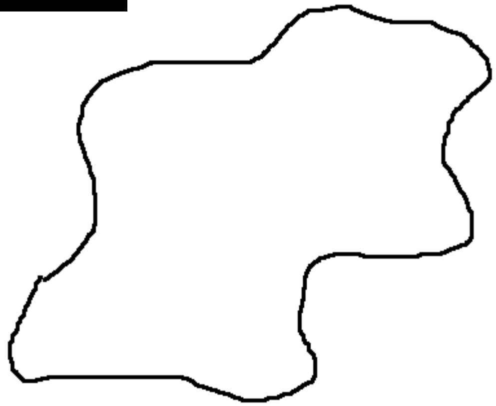


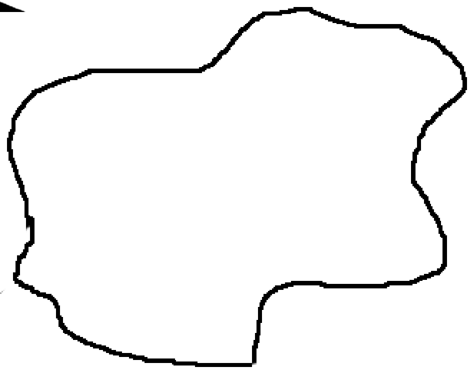


Differential invariants – an example









Differential invariants

DI’s are composed from higher-order derivatives of the boundary

DI’s are invariant to affine and even to projective transform but are extremely unstable

Semi-differential invariants

Motivation: avoiding high-order derivatives while preserving the locality

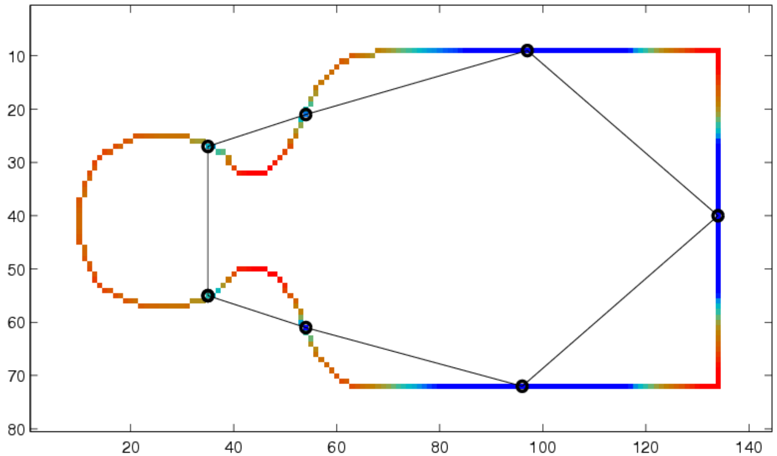
Decomposition of the object into parts, each part is then described by global invariants

The decomposition is often based on inflection points (they are affine- and projective-invariant)

$$\dot{x}\ddot{y} - \ddot{x}\dot{y} = 0$$

Dividing the object into invariant parts

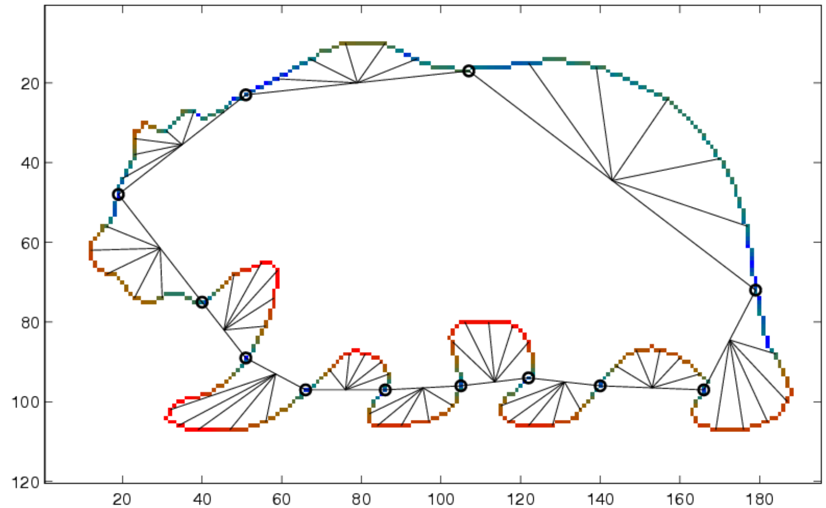
- Inflection points and centers of straight lines

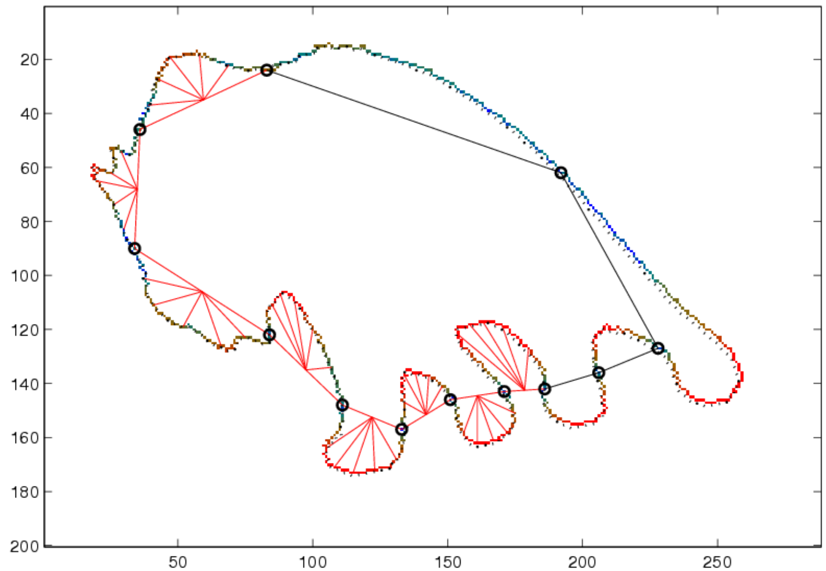


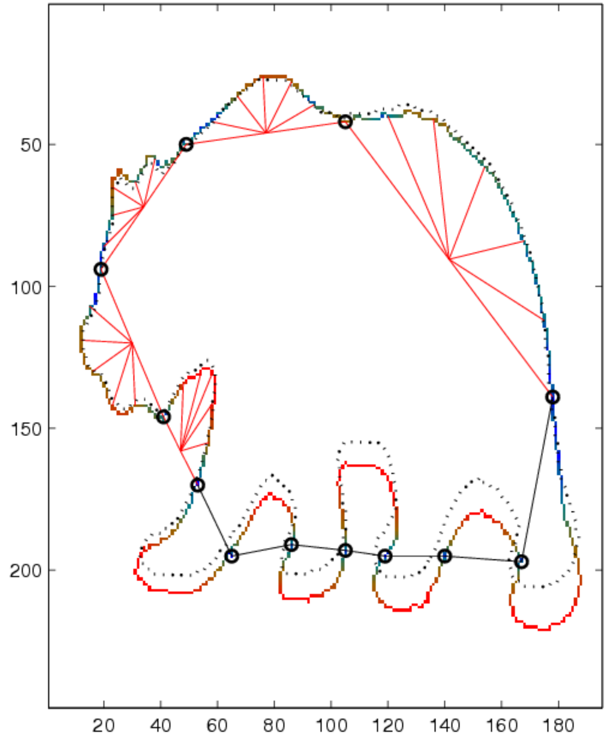
$$\dot{x}\ddot{y} - \ddot{x}\dot{y} = 0$$

- Computing invariants of each part

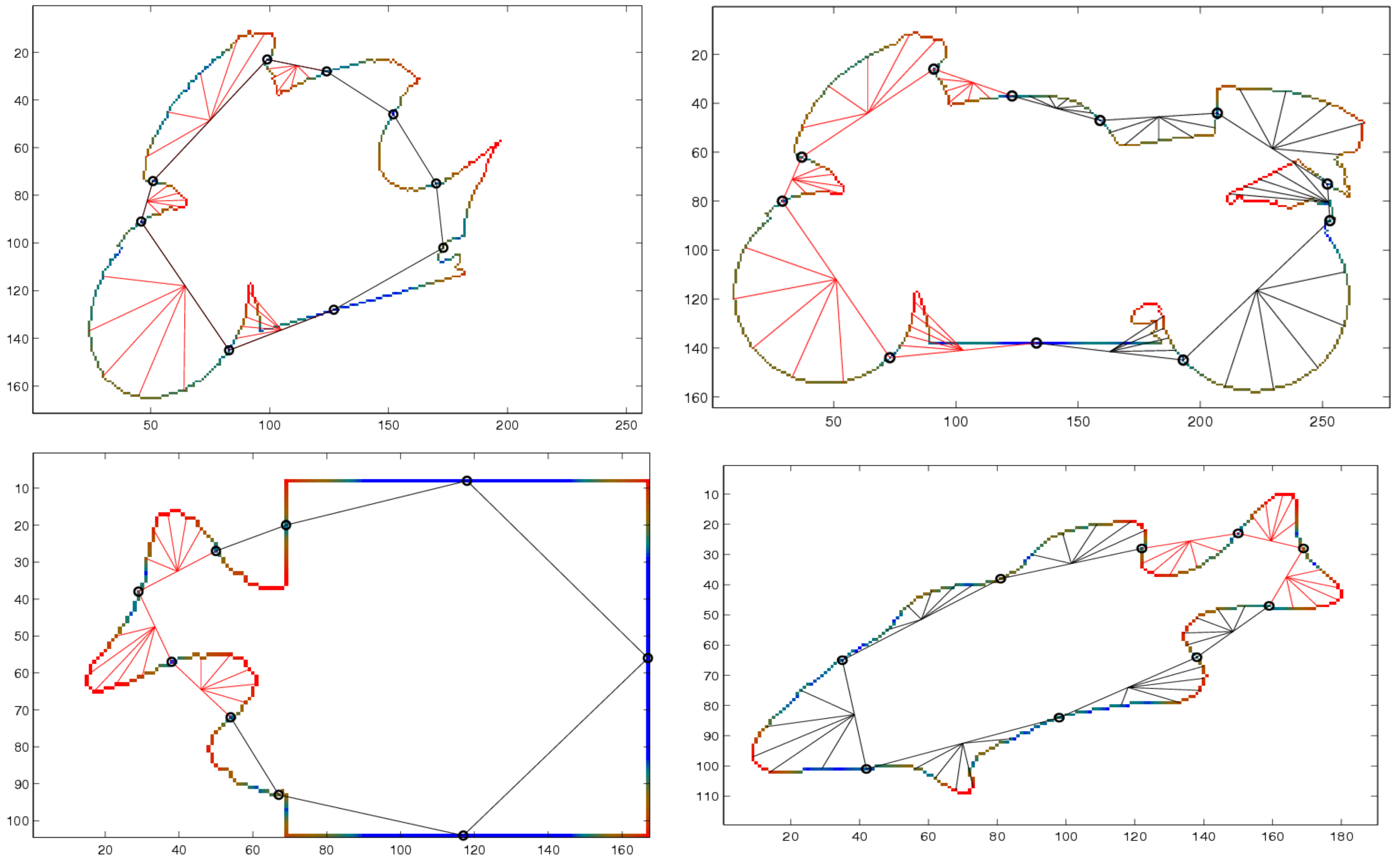
Affine-invariant radial vectors





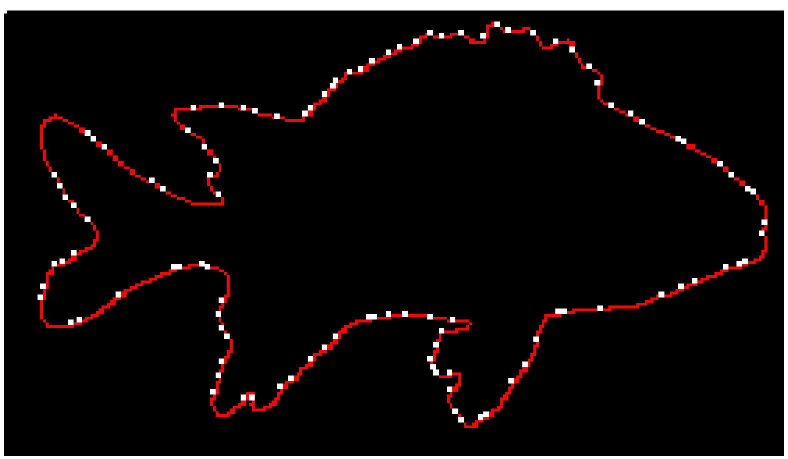


Recognition of occluded objects

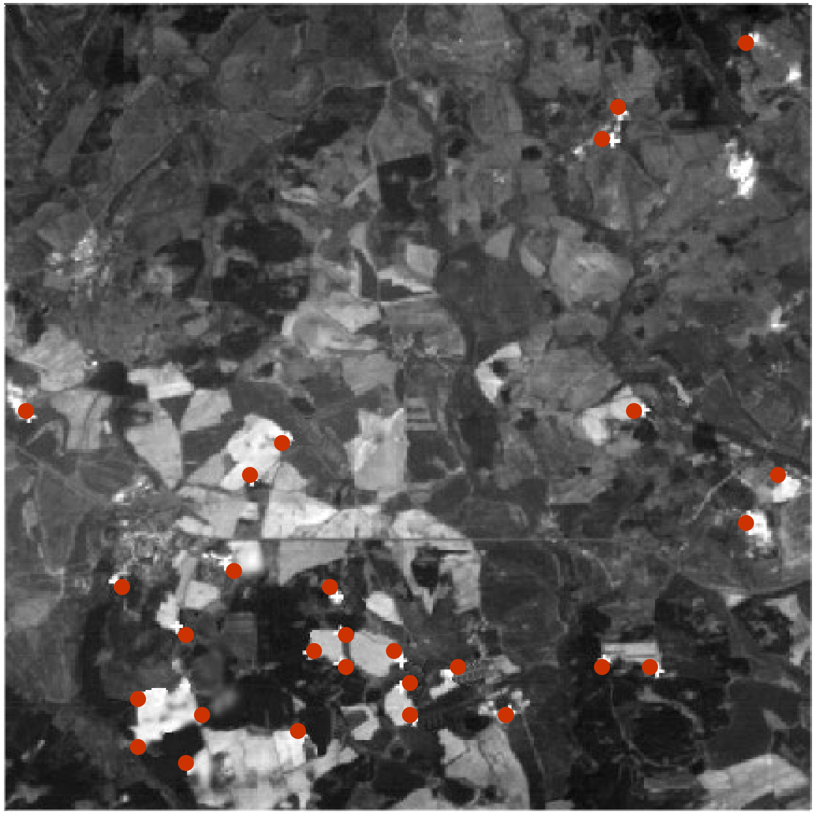


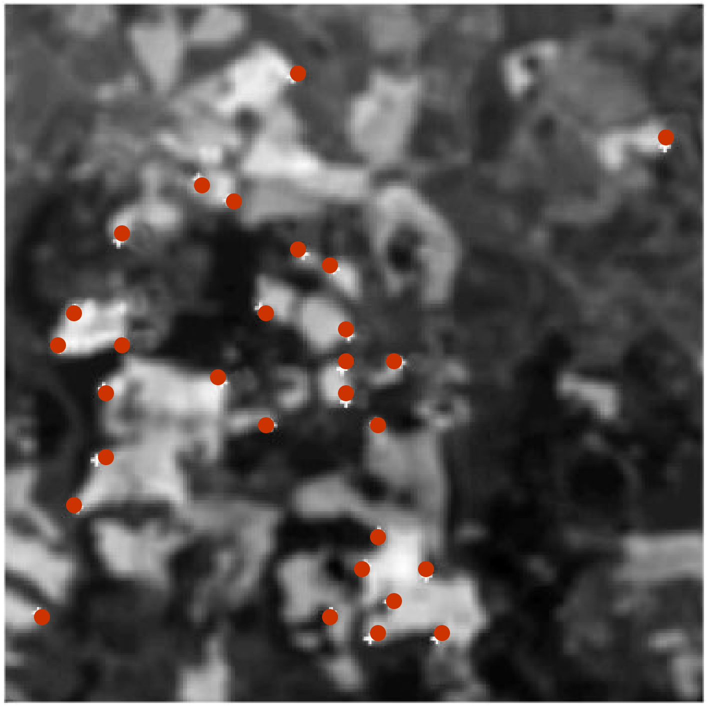
CSS – Curvature Scale Space

Evolution of inflection points at different scales



Local invariants of graylevel images





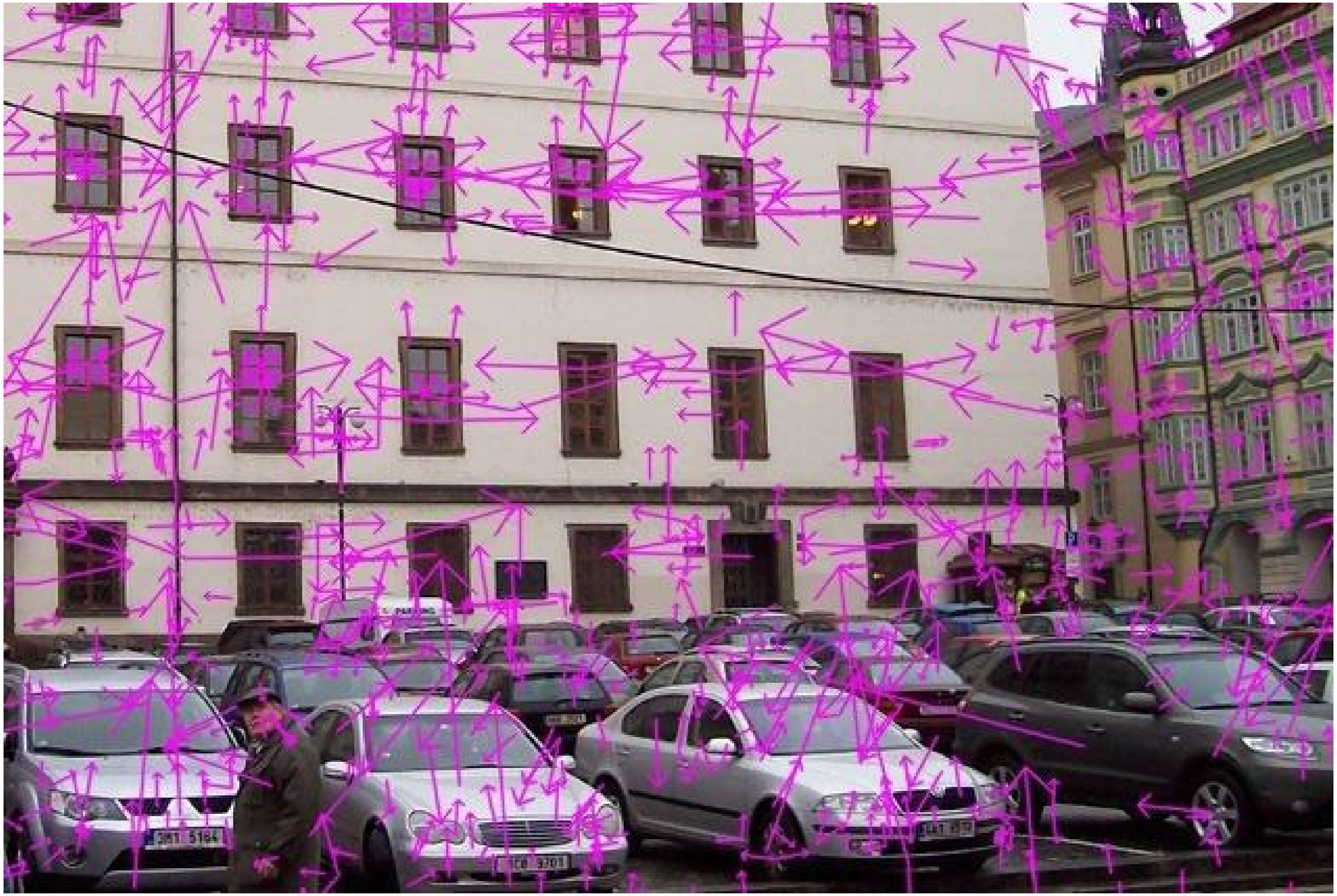
$$\min_{k,m} \ \text{distance}\left((v1_k, v2_k, v3_k, \dots), (\overline{v1}_m, \overline{v2}_m, \overline{v3}_m, \dots) \right)$$

SIFT (D.G. Lowe, 1999)

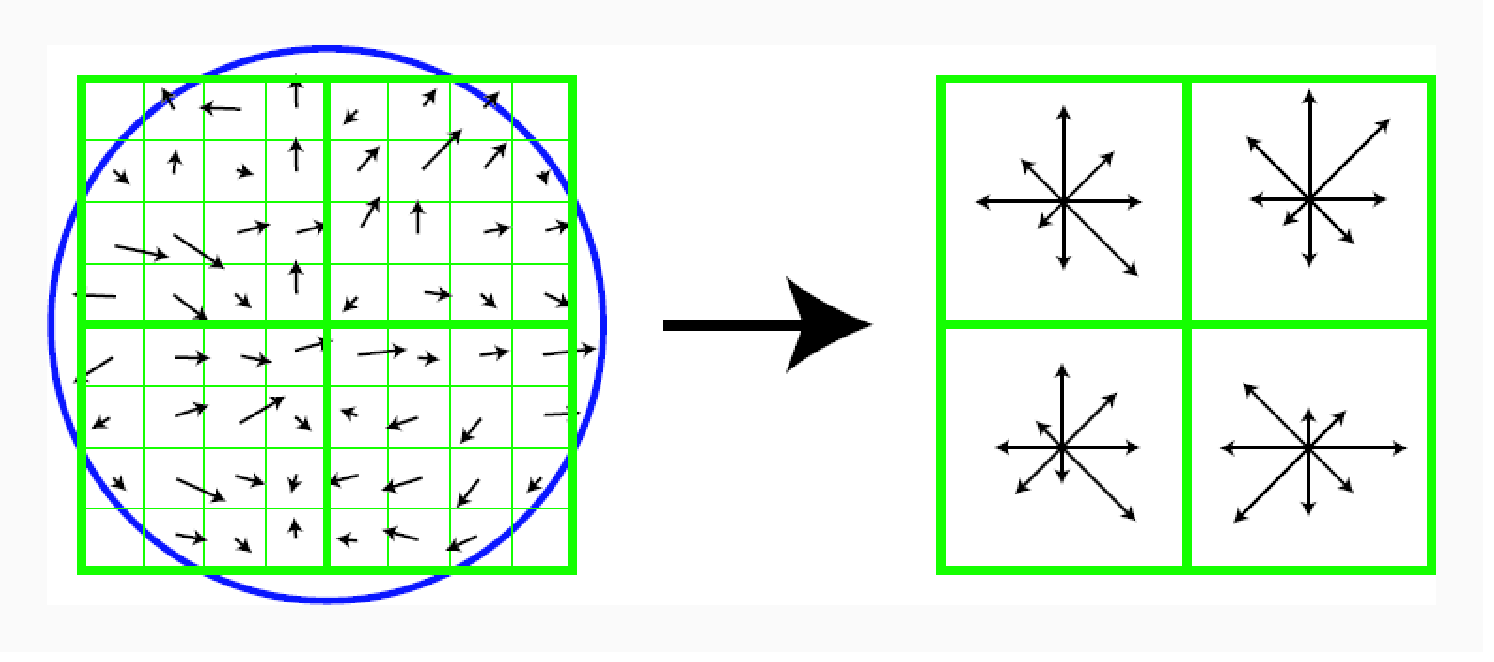
Find local extrema in the difference image

Delete local extrema with low contrast and those on edges

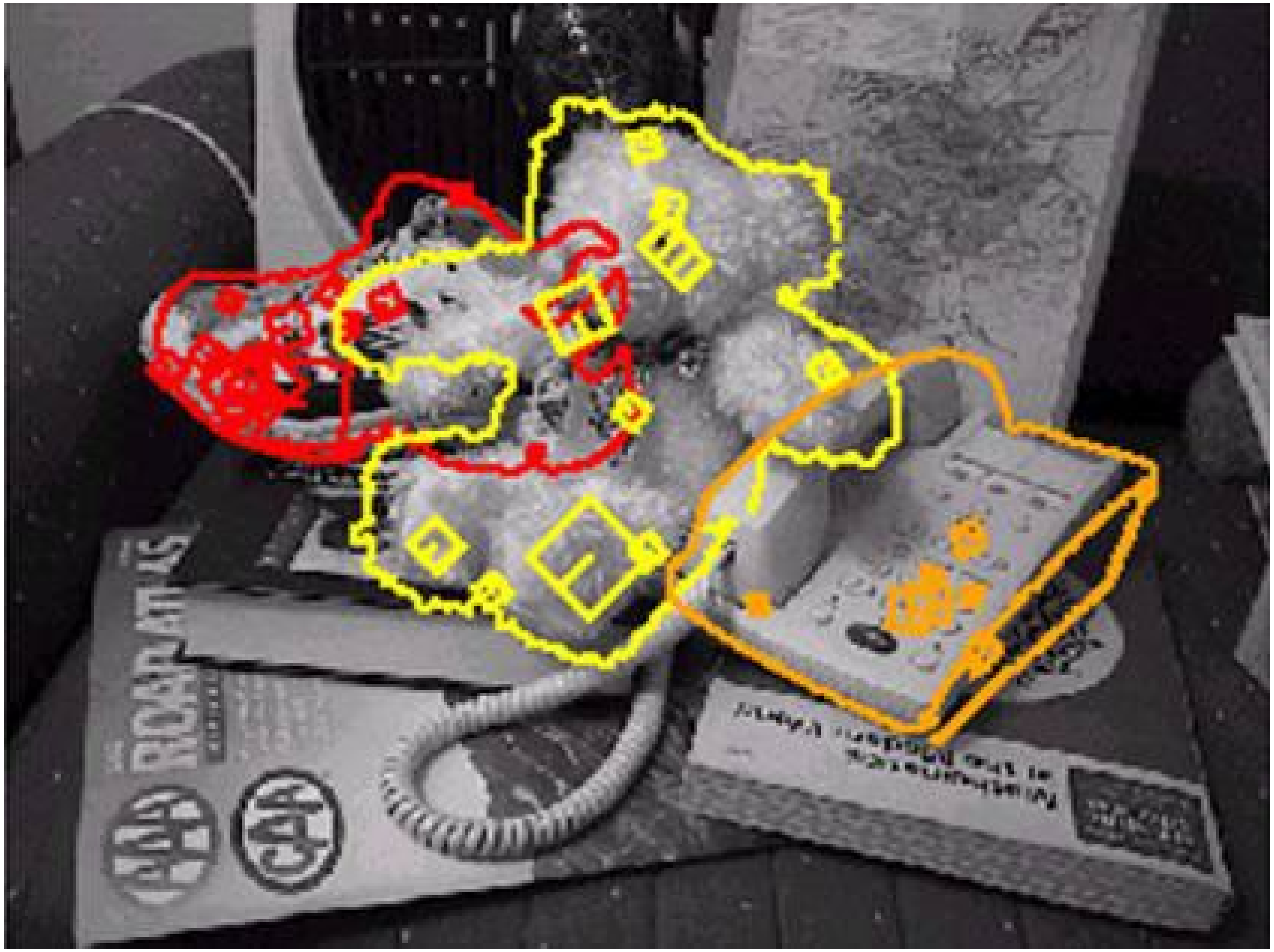
Gradient-based neighborhood



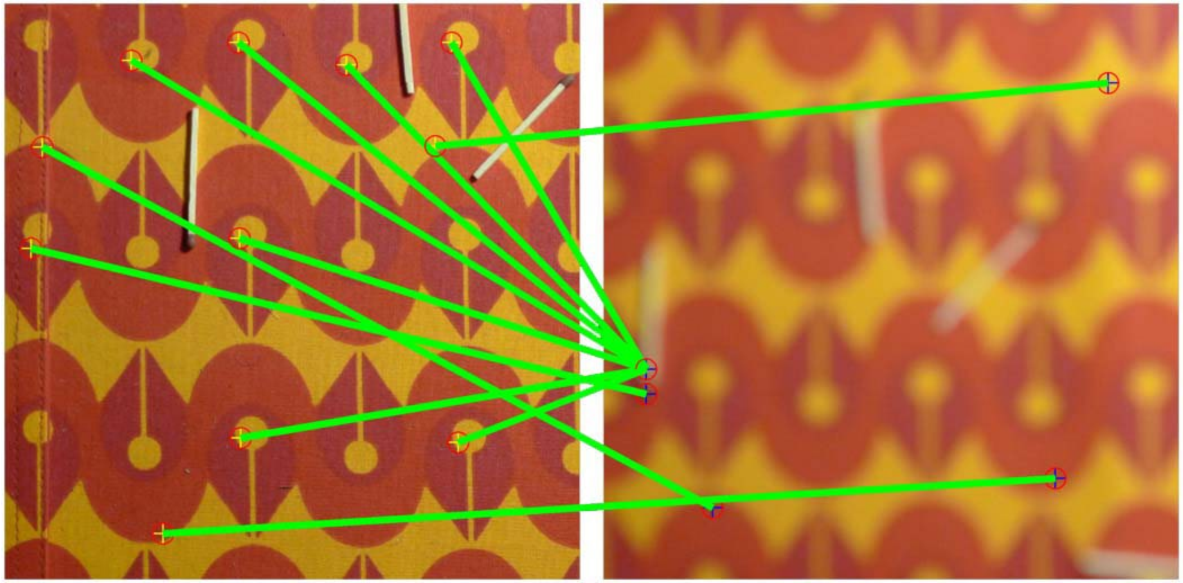
Feature vector: histogram of gradients In the neighborhood

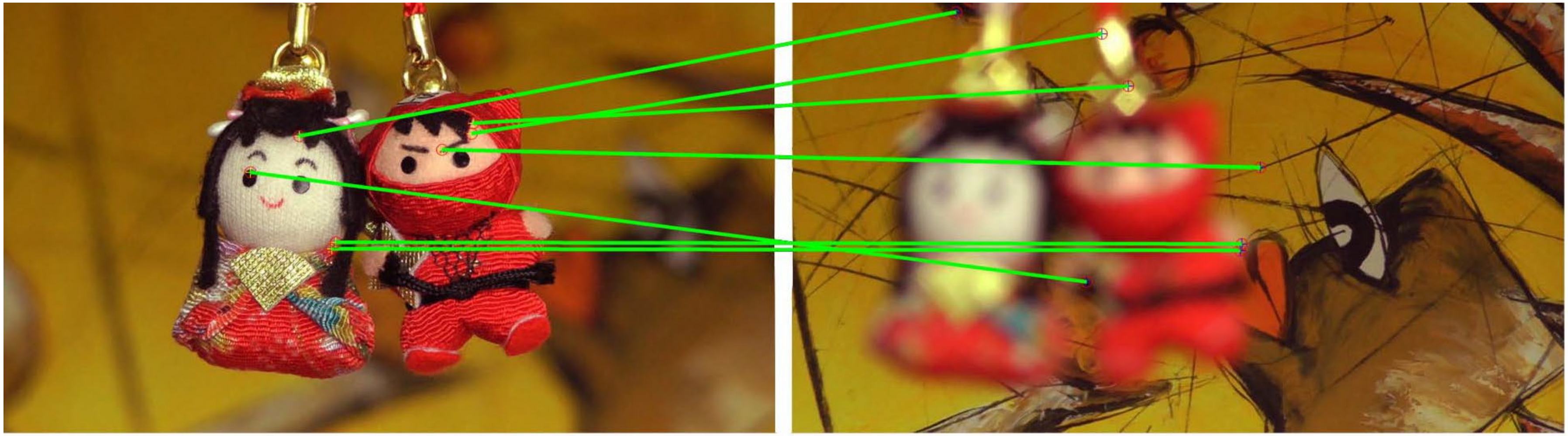


Recognition of occluded objects



Problems





Moment invariants

Moments are "projections" of the image function into a polynomial basis

$f(x, y)$ – piecewise continuous image function defined on bounded $\Omega \subset \mathcal{R} \times \mathcal{R}$

$\{\mathcal{P}_{pq}(x, y)\}$ – set of polynomials defined on Ω

$$M_{pq} = \int\int \mathcal{P}_{pq}(x, y) f(x, y) dx dy$$

Common types of moments

Geometric moments

$$m_{pq}^{(f)} = \int_{R^2} x^p y^q f(x,y) dx dy$$

Central moments

$$\mu_{pq}^{(f)} = \int_{R^2} (x - x_t)^p (y - y_t)^q f(x,y) dx dy$$

What are moment invariants?

Functions of moments, invariant to certain class of image degradations

- Rotation, translation, scaling
- Affine transform
- Convolution/blurring
- Combined invariants

Invariants to translation and scaling

Normalized central moments

$$\nu_{pq} = \frac{\mu_{pq}}{\mu_{00}^{w}} \qquad w = \frac{p+q}{2} + 1$$

Invariants to rotation

$$\phi_1 = \mu_{20} + \mu_{02}$$

$$\phi_2 = (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2$$

$$\phi_3 = (\mu_{30} - 3\mu_{12})^2 + (3\mu_{21} - \mu_{03})^2$$

$$\phi_4 = (\mu_{30} + \mu_{12})^2 + (\mu_{21} + \mu_{03})^2$$

Thank you !

Any questions ?