Image Restoration

Image Restoration

- Diffusion
- Denoising
- Deconvolution
- Super-resolution
- Tomographic Reconstruction

Diffusion Term

- Consider only the regularization term

$$F(u) = \int_\Omega |\nabla u|^2 dx$$

- E-L equation: (Laplace equation)

$$F'(u) = -\Delta u = 0$$

- Steepest Descent:

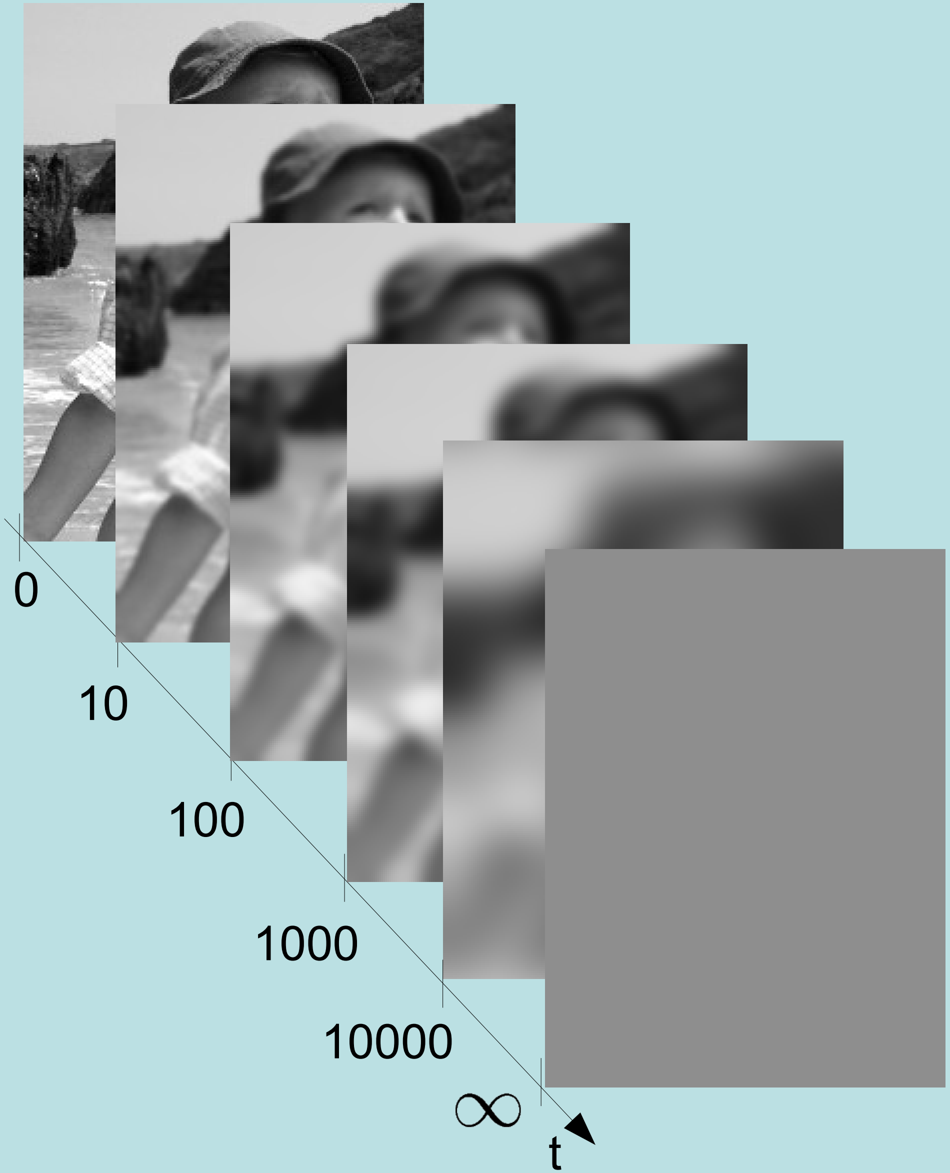
$$u_{k+1} = u_k + \alpha \Delta u$$

Evolution of Laplace's Equation

$$F(u) = \int_\Omega |\nabla u|^2 dx$$

$$u_t = \Delta u$$

$$\mathbf{u}_{k+1} = \mathbf{u}_k - \alpha \mathbf{C} \mathbf{u}_k$$

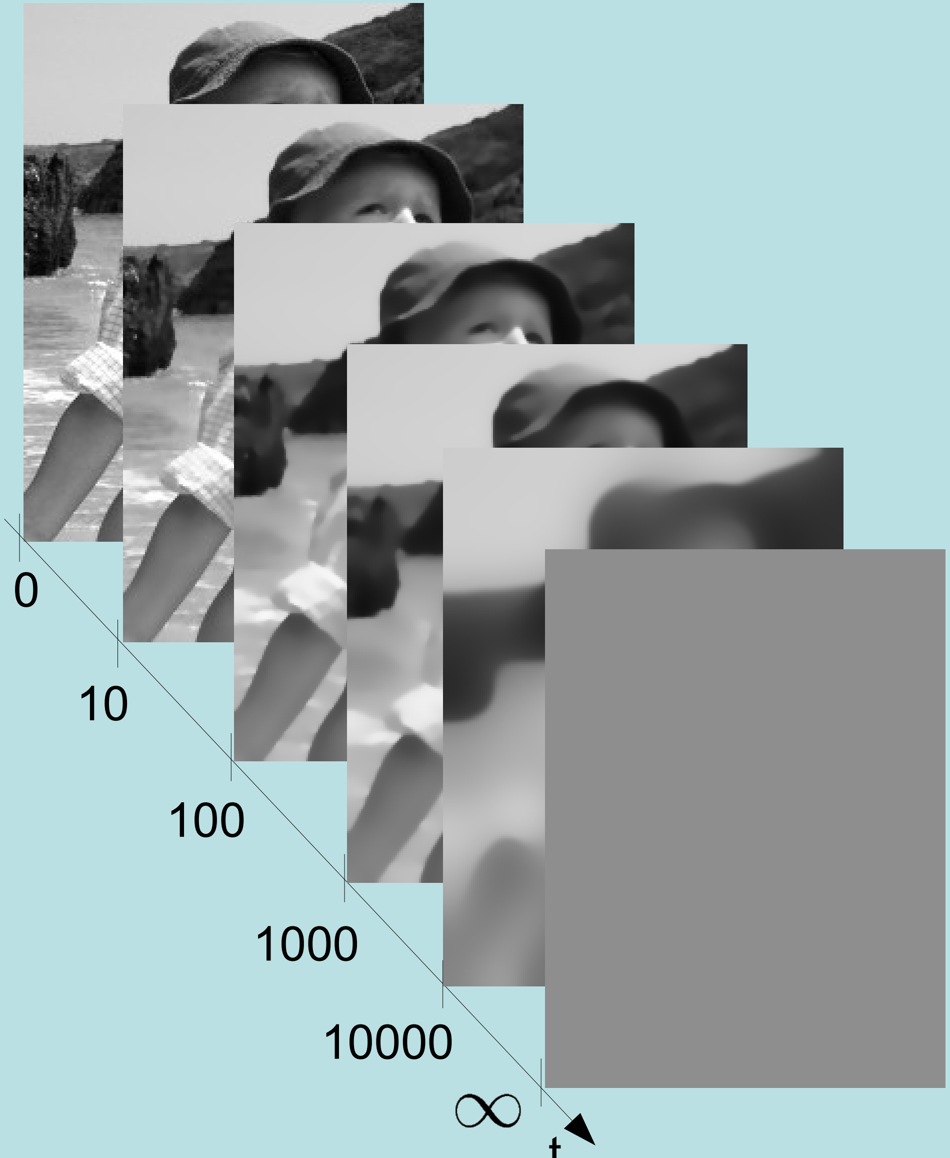


Evolution of TV Equation

$$F(u) = \int_\Omega |\nabla u| dx$$

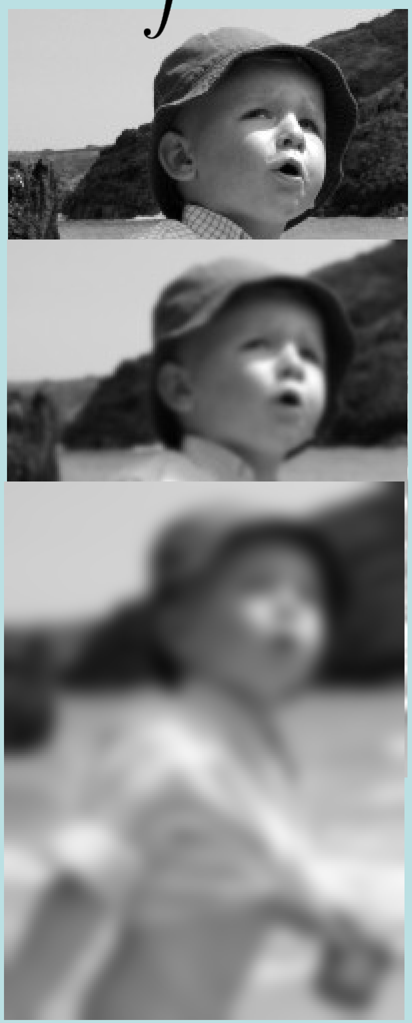
$$u_t = \mathrm{div}\left(\frac{\nabla u}{|\nabla u|}\right)$$

$$\mathbf{u}_{k+1} = \mathbf{u}_k - \alpha \mathbf{L}_{\nabla \mathbf{u}_k} \mathbf{u}_k$$

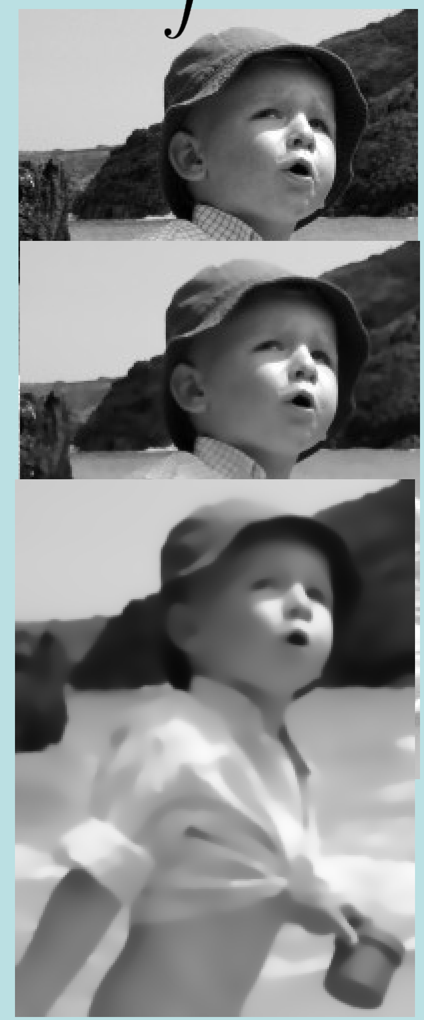


Isotropic & Anisotropic Diffusion

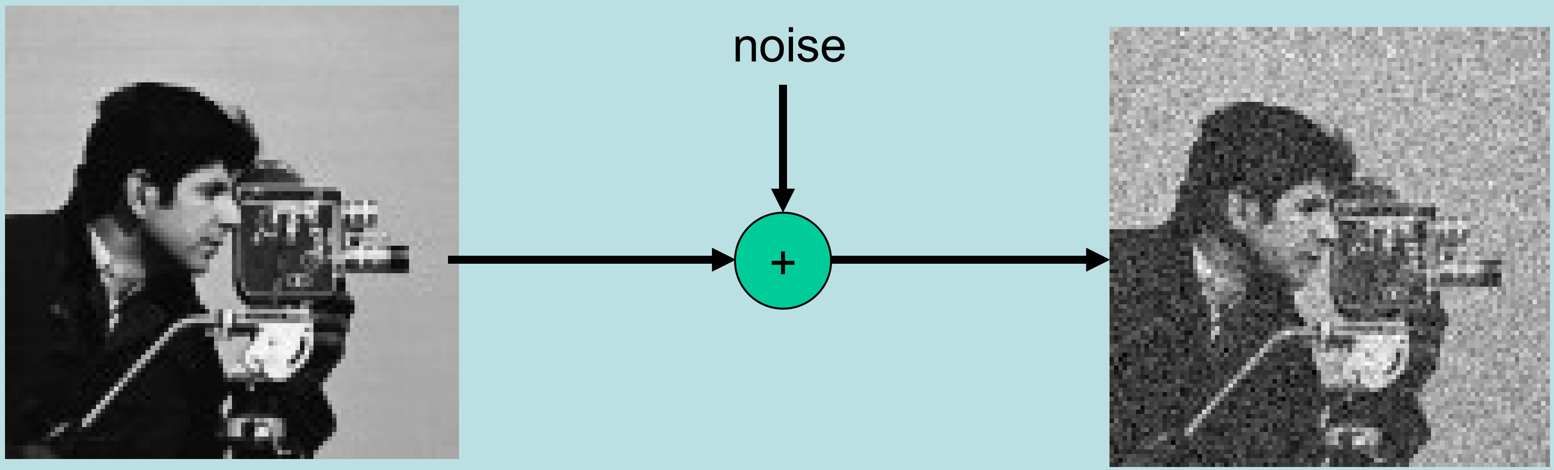
$$\min \int |\nabla u|^2$$



$$\min \int |\nabla u|$$



Acquisition model with noise



original image

acquired images

$$u(x) \quad + \quad n(x) \quad = \quad z(x)$$

Denoising

- Acquisition model

$$z = u + n \qquad n \ldots N(0, \sigma_n^2)$$

- Minimization problem

$$F(u) = \frac{1}{2}\int_\Omega |z-u|^2 dx + \lambda \int_\Omega |\nabla u|^2 dx$$

Data term

Weighting parameter

Regularization term

Equivalent Formulations

$$\min \left\{ \frac{1}{2} \int_\Omega |z - u|^2 dx + \lambda \int_\Omega |\nabla u|^2 dx \right\}$$

- Constraint minimization

$$\min \int |\nabla u|^2 \quad \text{subject to } \|z - u\|^2 = \sigma_n^2$$

- Maximum A Posteriori (MAP) estimate

$$\min \{ -\log p(u|z) \}$$

Bayes' Theorem:

$$p(u|z) = \frac{p(z|u)p(u)}{p(z)}$$

- Denoising functional:

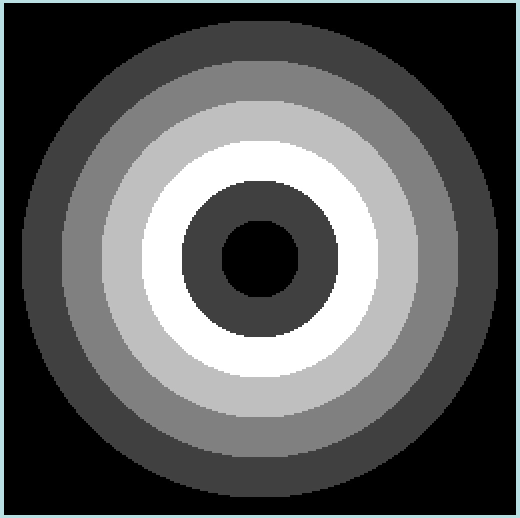
$$F(u) = \frac{1}{2}\int_\Omega |z-u|^2 dx + \lambda \int_\Omega |\nabla u|^2 dx$$

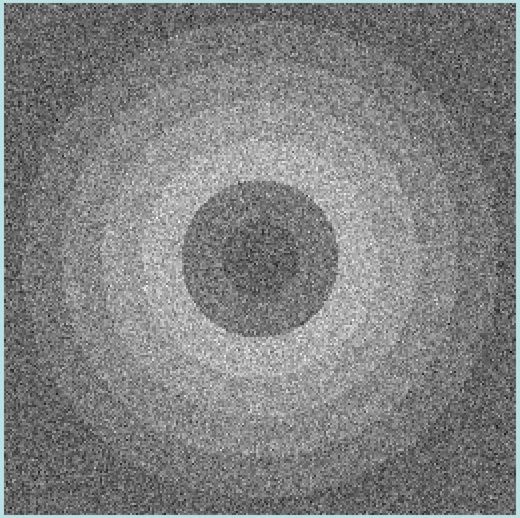
- E-L equation:

$$F'(u) = (z-u) - \lambda \Delta u = 0$$

- Discrete solution:
  – Set of linear equations

$$(\mathbf{I} + \lambda \mathbf{L})\mathbf{u} = \mathbf{z}$$



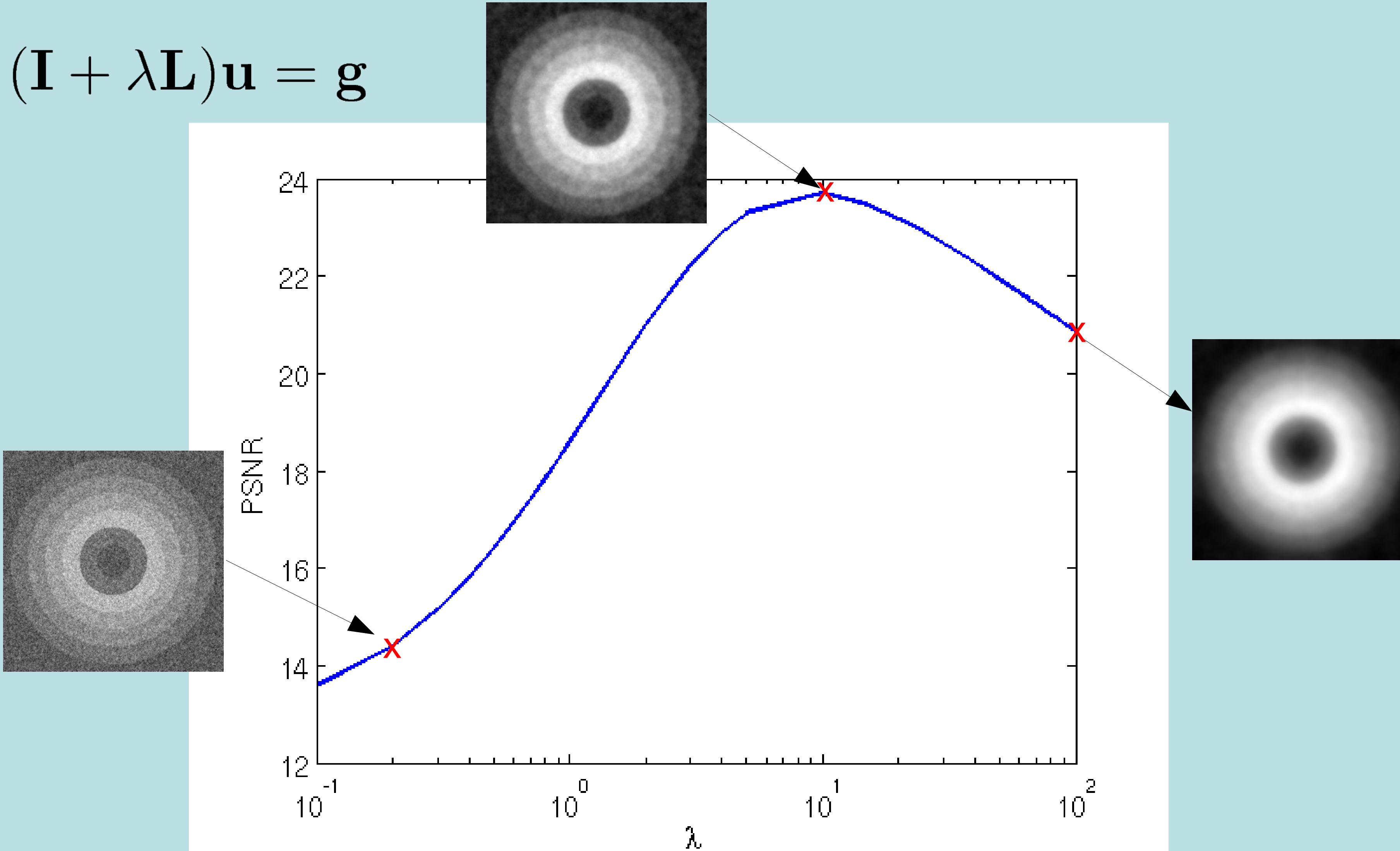


Original image u

noisy image z

Regularization Weight "λ"

$$(\mathbf{I} + \lambda \mathbf{L})\mathbf{u} = \mathbf{g}$$



Regularization

$$F(u) = \frac{1}{2}\int_\Omega |z-u|^2 dx + \lambda \int_\Omega \phi(|\nabla u|)dx$$

$$F'(u) = (z-u) - \lambda \,\mathrm{div}\left(\frac{\phi'(|\nabla u|)}{|\nabla u|}\nabla u\right) = 0$$

$$\phi(s) = s^2$$

- L^2 norm … Tichonov

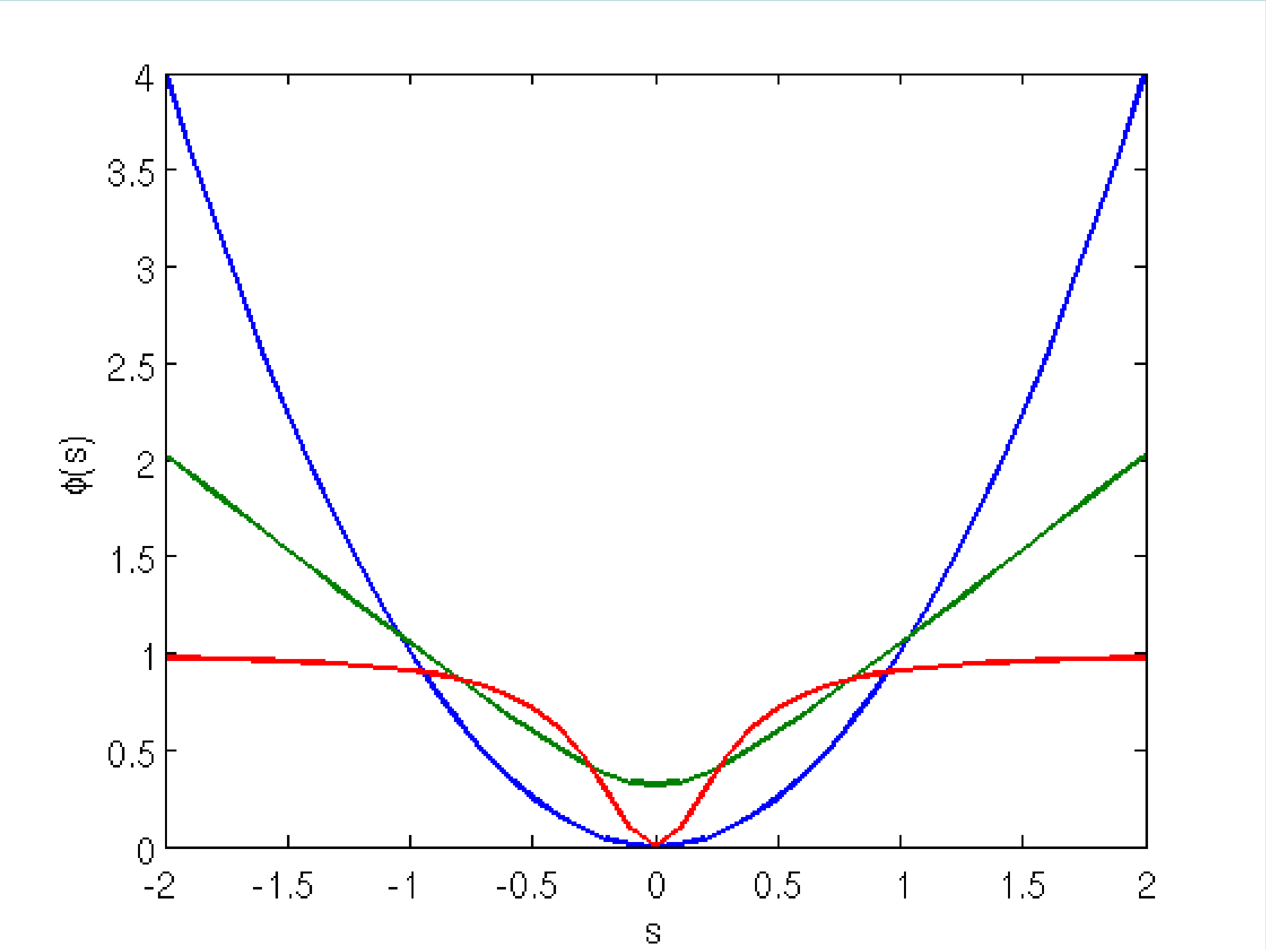
$$\phi(s) = \sqrt{\epsilon + s^2}$$

- L^1 norm … Total Variation

$$\phi(s) = \frac{s^2}{\epsilon + s^2}$$

- Nonconvex

Regularization

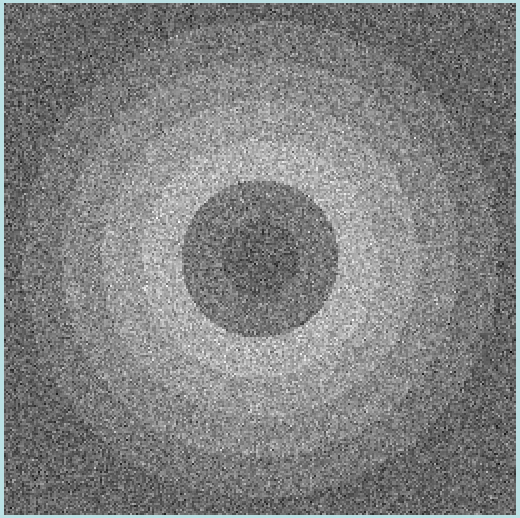


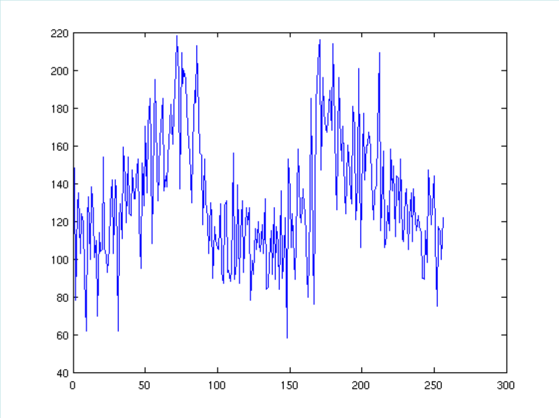
$$\phi(s) = s^2$$

$$\phi(s) = \sqrt{\epsilon + s^2}$$

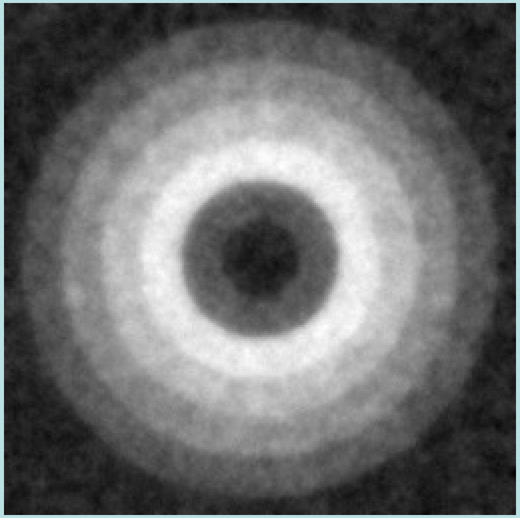
$$\phi(s) = \frac{s^2}{\epsilon + s^2}$$

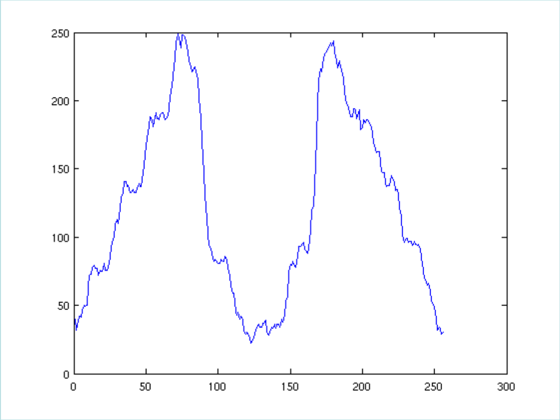
Noisy input image



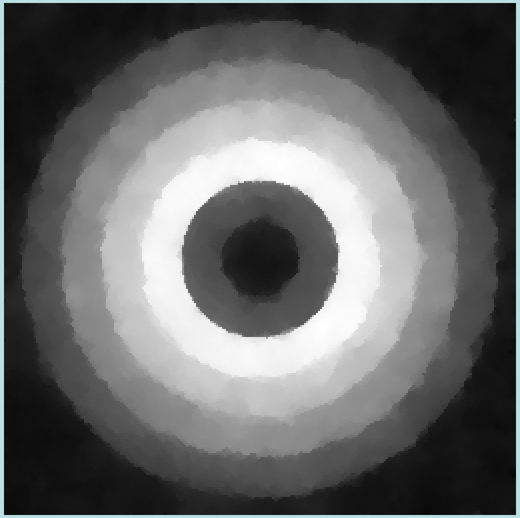


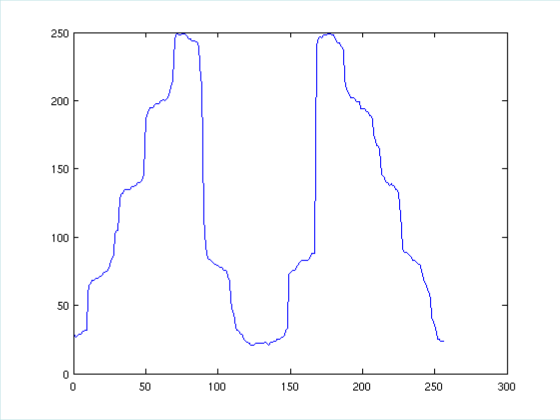
$$\phi(s) = s^2$$





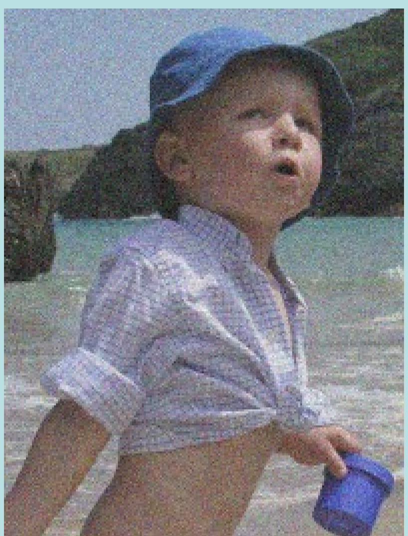
$$\phi(s) = \sqrt{\epsilon + s^2}$$





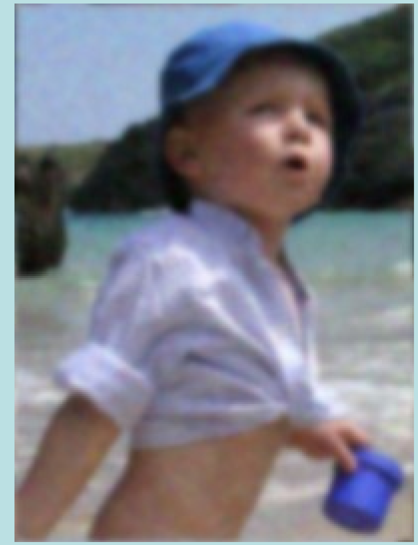
original

noisy



$\phi(s) = 0$

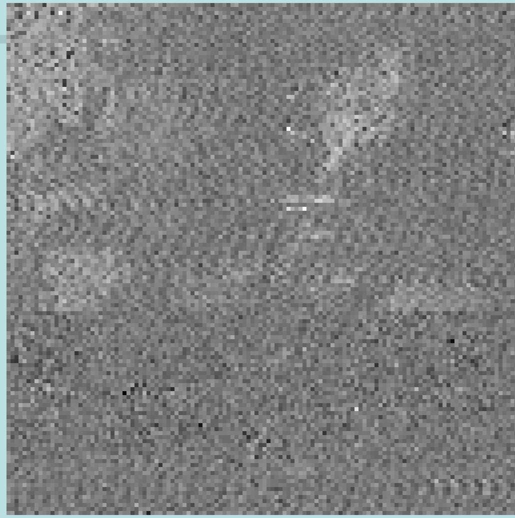
$\phi(s) = s^2$



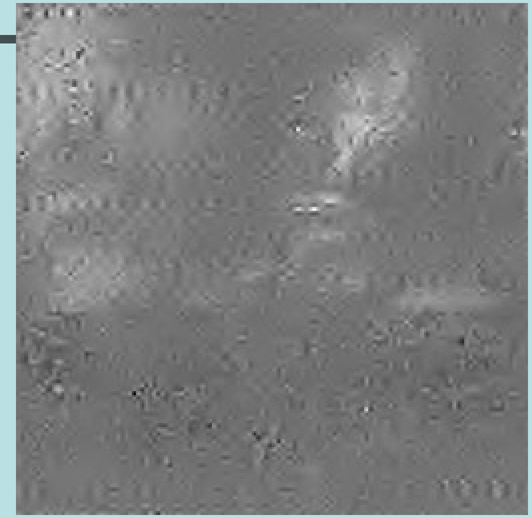
$\phi(s) = \sqrt{\epsilon + s^2}$

$\phi(s) = \frac{s^2}{\epsilon + s^2}$

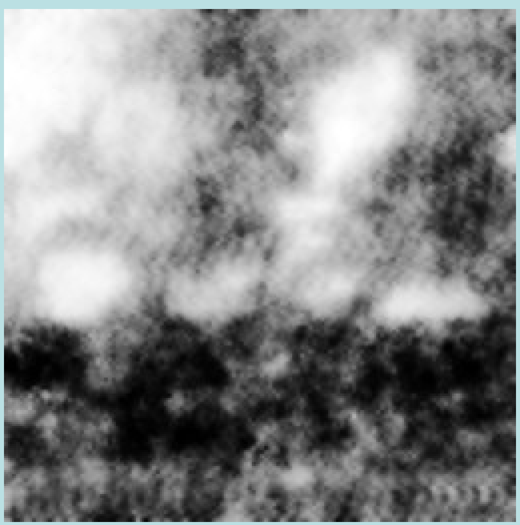
Anisotropic Denoising



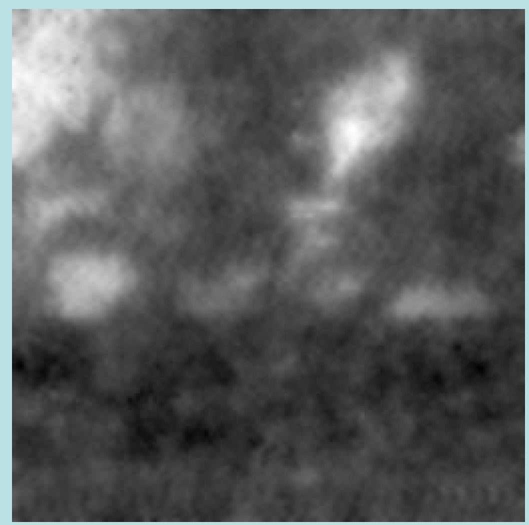
noisy BEEM image



wavelet-based denoising

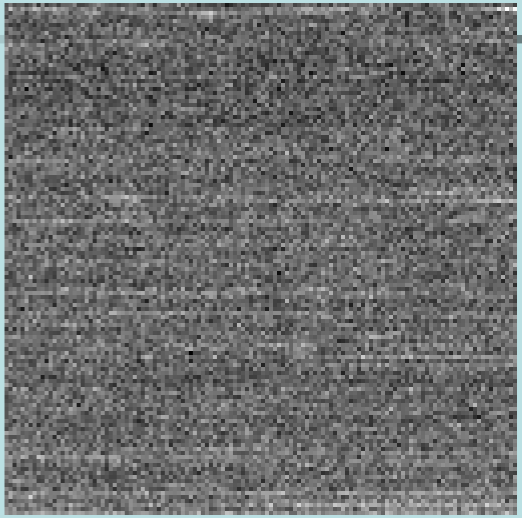


TV + histogram equalization

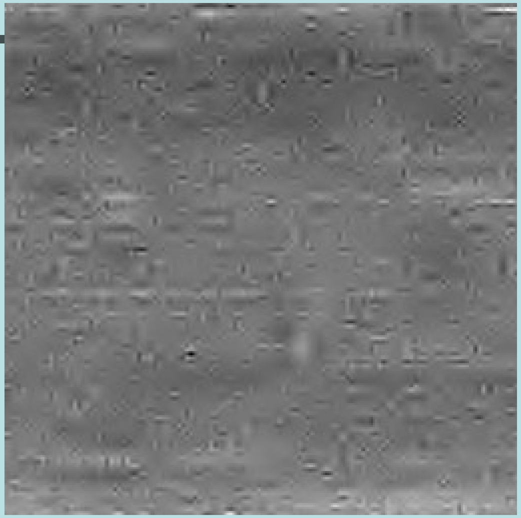


TV-based denoising

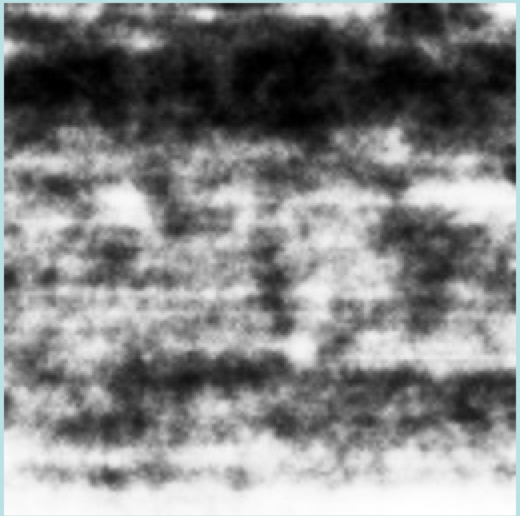
Anisotropic Denoising



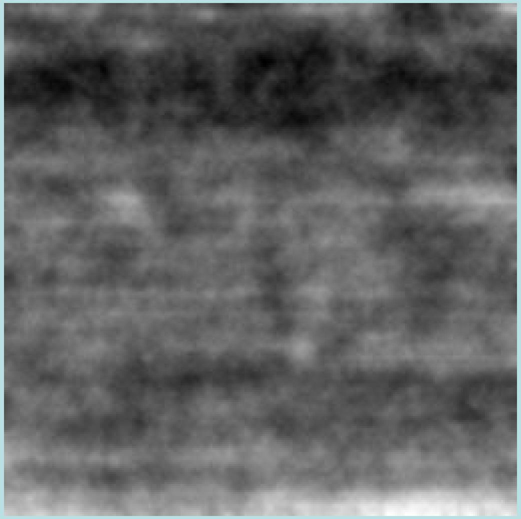
noisy BEEM image



wavelet-based denoising

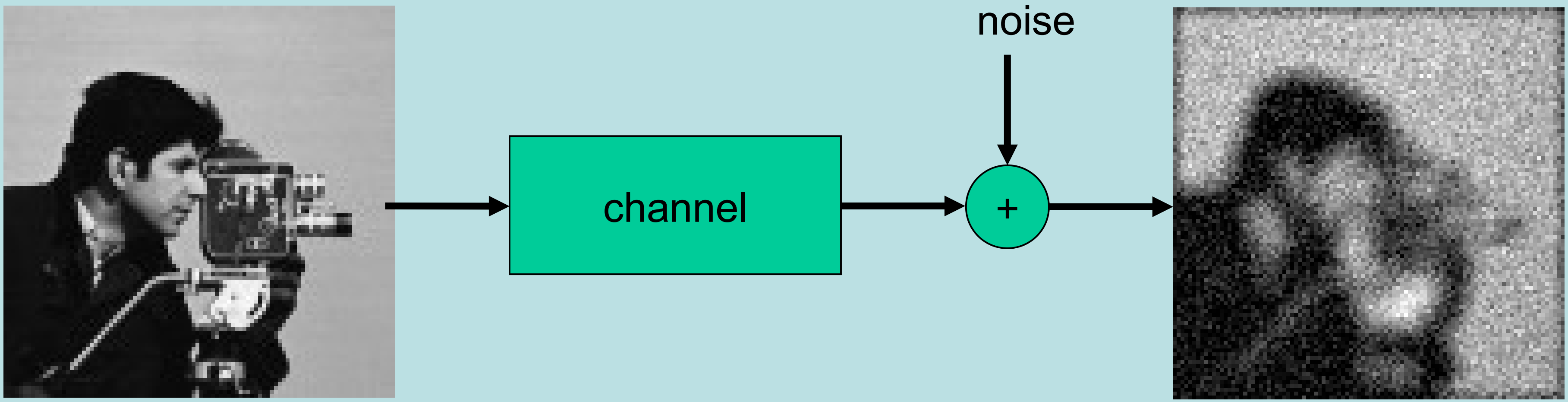


TV + histogram equalization



TV-based denoising

Acquisition model with blur



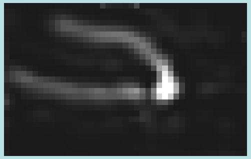
original image

acquired image

$$[u * h](x) + n(x) = z(x)$$

Motion Blur

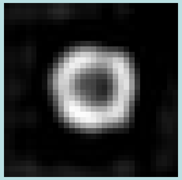
*



=

Out-of-focus Blur

*



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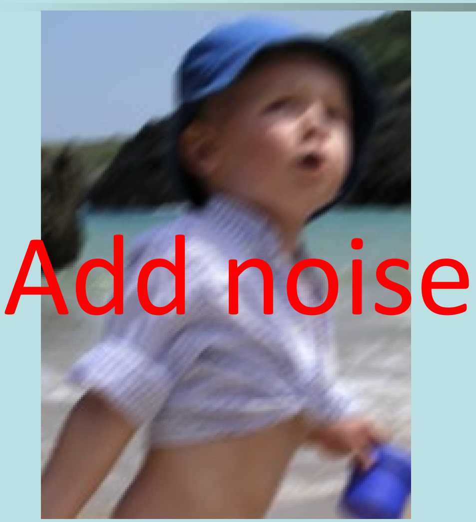
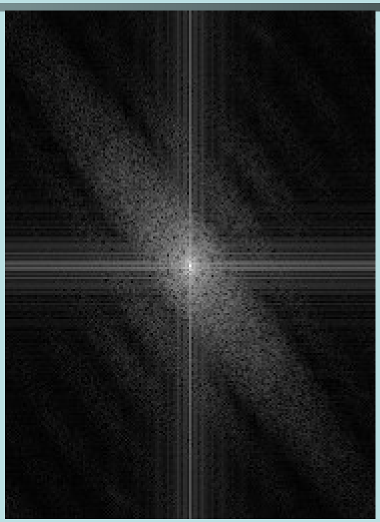
Inverse Filter

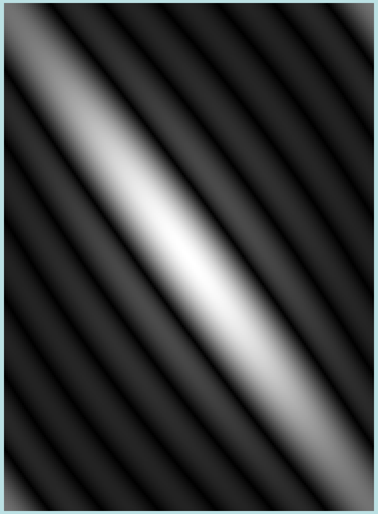


$z(x)$



$Z(\omega)$

$h(x)$



$H(\omega)$

$$\tilde{U}(\omega) = \frac{Z(\omega)}{H(\omega)}$$

equivalent

$$F(u) = \|z - h * u\|^2$$

Prove it!

Wiener Filter

$z(x)$

Replace by a constant

$$\tilde{\boldsymbol{U}} = \frac{\boldsymbol{H}^*}{|\boldsymbol{H}|^2 + \frac{S_n}{S_u}} \boldsymbol{Z}$$

$$\tilde{\boldsymbol{U}} = \frac{\boldsymbol{H}^*}{|\boldsymbol{H}|^2 + \lambda} \boldsymbol{Z}$$

equivalent

$$F(u) = \|z - h * u\|^2 + \lambda \|u\|^2$$

$$\tilde{u} = \arg\min_u F(u)$$

$\lambda = 0.001$

$\lambda = 0.1$

Deblurring (Deconvolution)

- Acquisition mode

$$z = (h * u) + n \qquad \begin{array}{l} n \ldots N(0, \sigma_n^2) \\ h \ldots \text{convolution kernel} \end{array}$$

- Minimization problem

$$F(u) = \frac{1}{2} \int_\Omega |z - h * u|^2 dx + \lambda \int_\Omega \phi(|\nabla u|) dx$$

Data term

Weighting parameter

Regularization term

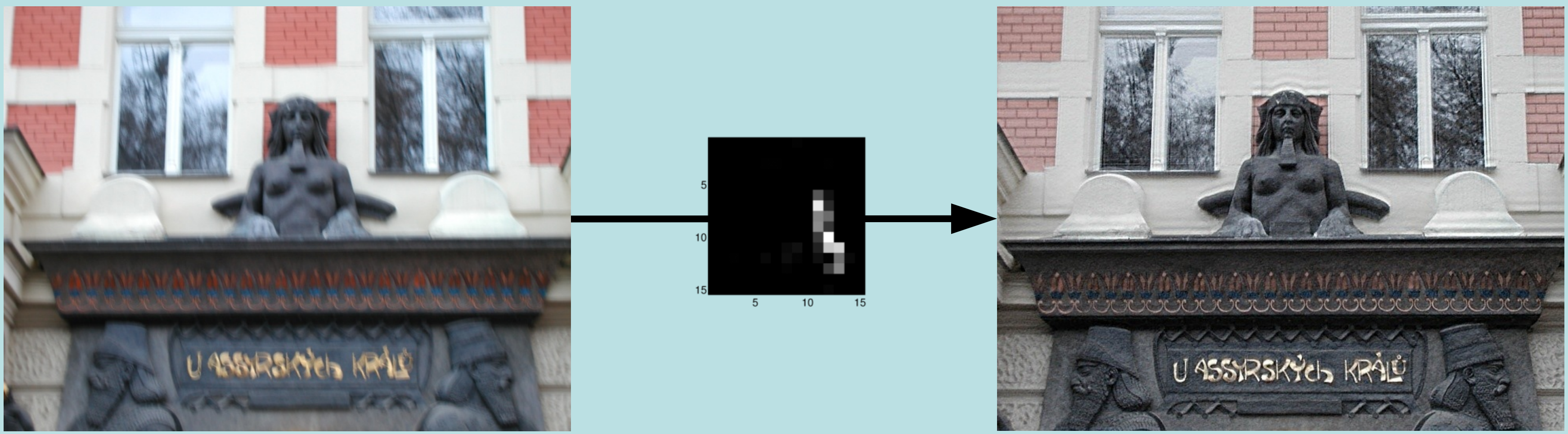
- E-L equation:

$$F'(u) = h \circledast (z - h * u) - \lambda \mathrm{div}\left(\frac{\phi'(|\nabla u|)}{|\nabla u|}\nabla u\right) = 0$$

- Discrete solution:
  - Set of linear equations

$$(\mathbf{H}^T\mathbf{H} + \lambda \mathbf{L}_{\nabla u})\mathbf{u} = \mathbf{H}^T\mathbf{z}$$

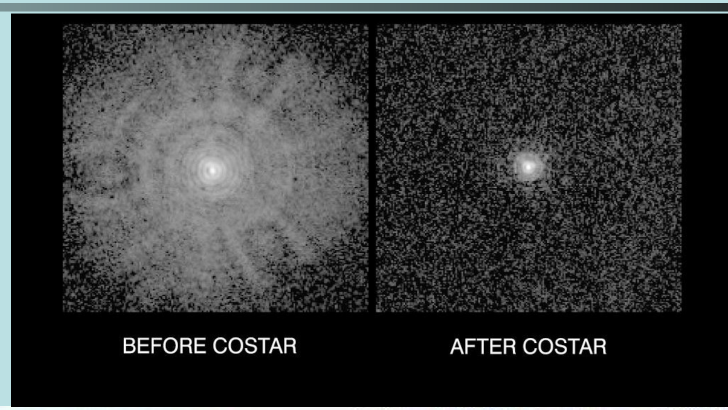
Long-time Exposure



How tackle the blind case?

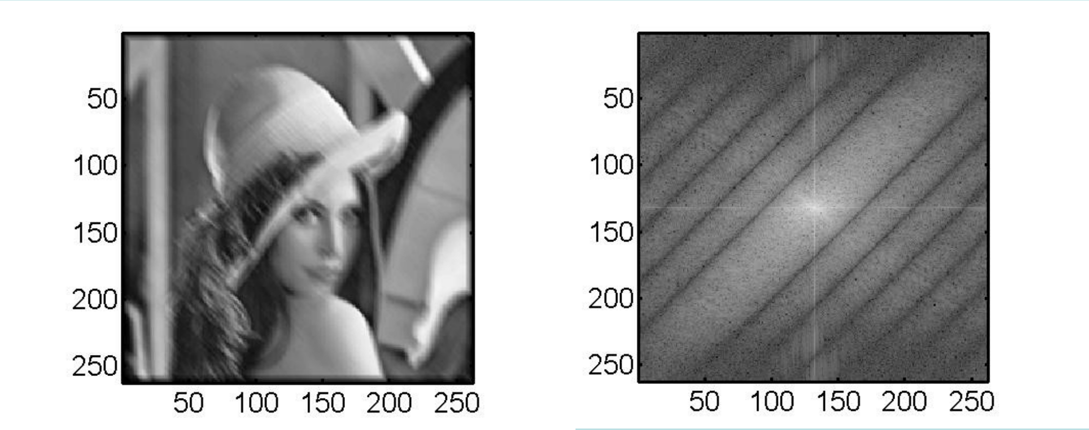
- When the blur kernel $h(x)$ is not known
- Estimate blur by other means

Blur estimation from point source

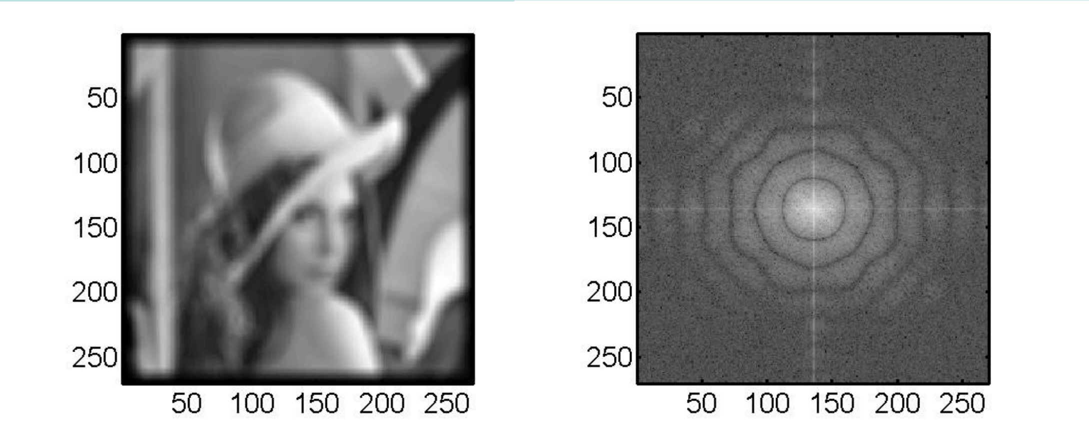


Blur estimation from spectra

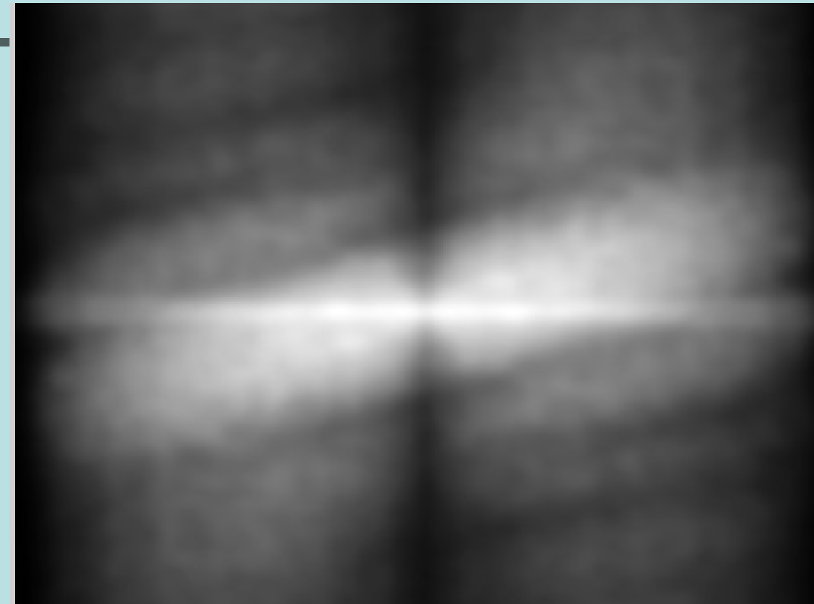
- Motion blur

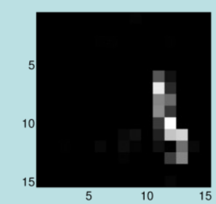


- Out-of-focus blur



- Works only for precise line and cylinder!





True PSF is not a precise line

How tackle the blind case?

- When the blur kernel $h(x)$ is not known
- Estimate blur by other means

- One is tempted to:
  1) Add blur regularization
  2) Perform alternating minimization

Alternating Minimization

$$\min_{u,h} F(u,h) = \min_{u,h} \frac{1}{2}\|u * h - z\|^2 + \lambda Q(u) + \gamma R(h)$$

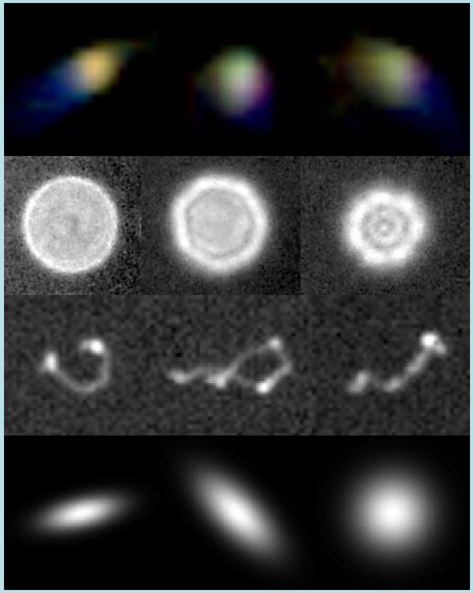
Blur Regularization term

- Alternate between two steps:
  1) $\tilde{u} = \arg\min_{u} F(u, \tilde{h})$
  2) $\tilde{h} = \arg\min_{h} F(\tilde{u}, h)$

Blur regularization

$$\min_{u,h} F(u,h) = \min_{u,h} \frac{1}{2}\|u * h - z\|^2 + \lambda Q(u) + \gamma R(h)$$

- Blur has different shape
  - Compact support
  - Non-negative
  - Preserve energy



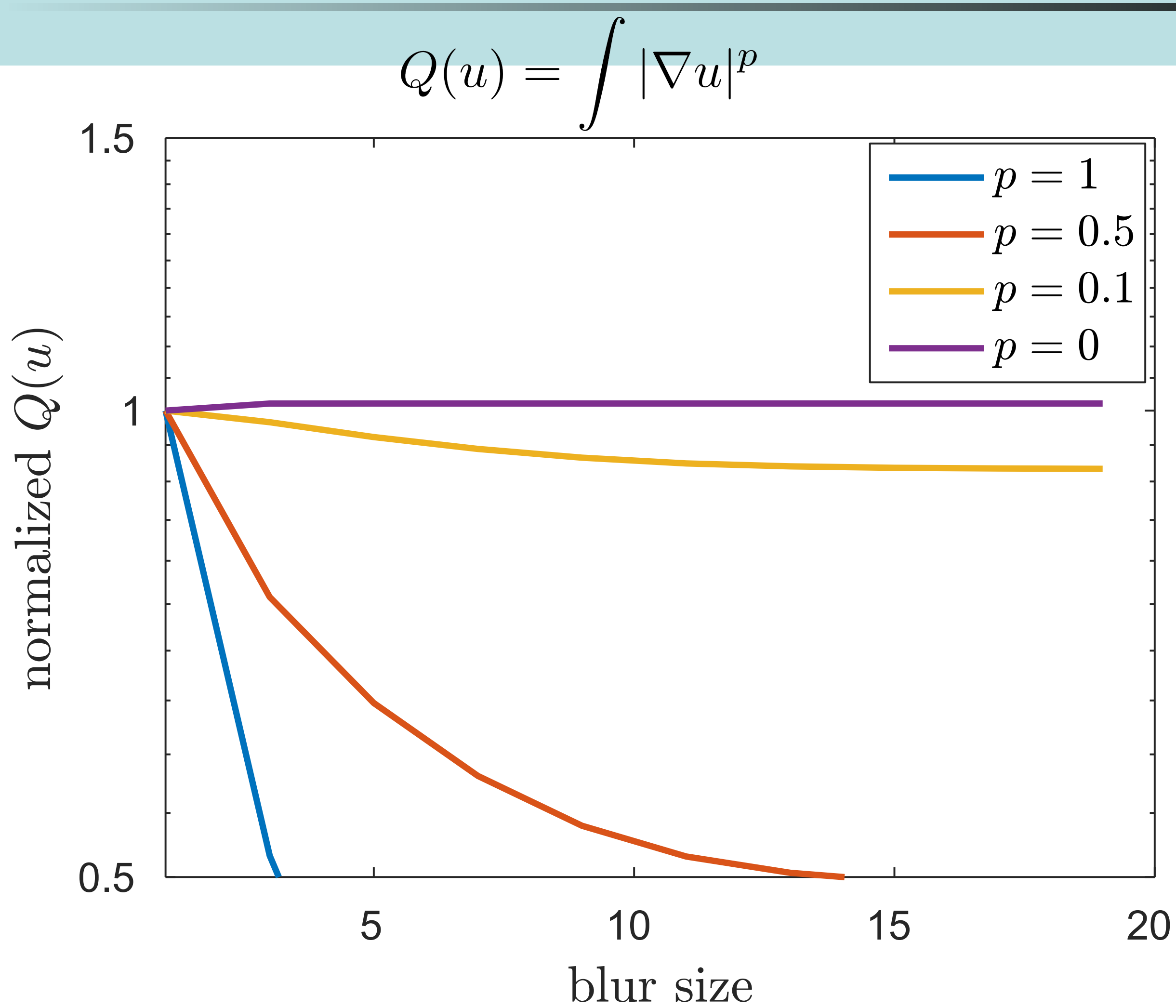
“No-blur” solution

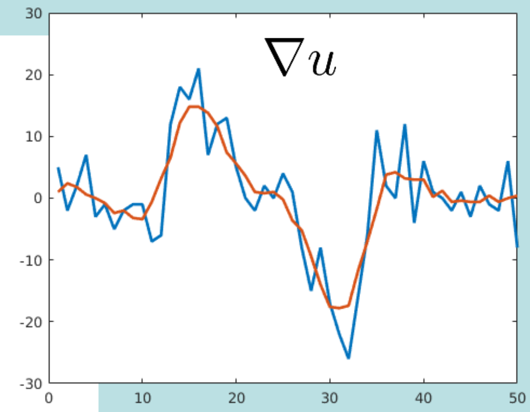
$$\min_{u,h} F(u,h) = \min_{u,h} \frac{1}{2}\|u * h - z\|^2 + \lambda Q(u) + \gamma R(h)$$

- Both image and blur regularization do not penalize the solution:

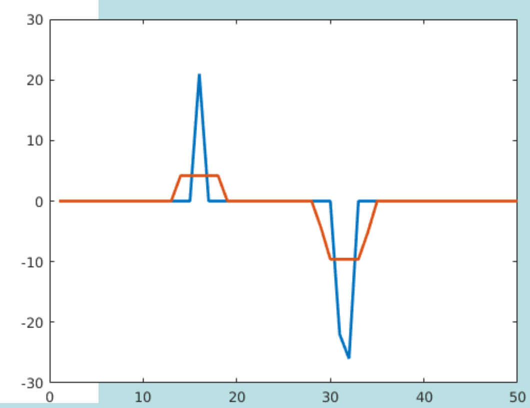
$$\tilde{u}(x) = z(x), \quad \tilde{h}(x) = \delta(x)$$

Ragularization favors blur



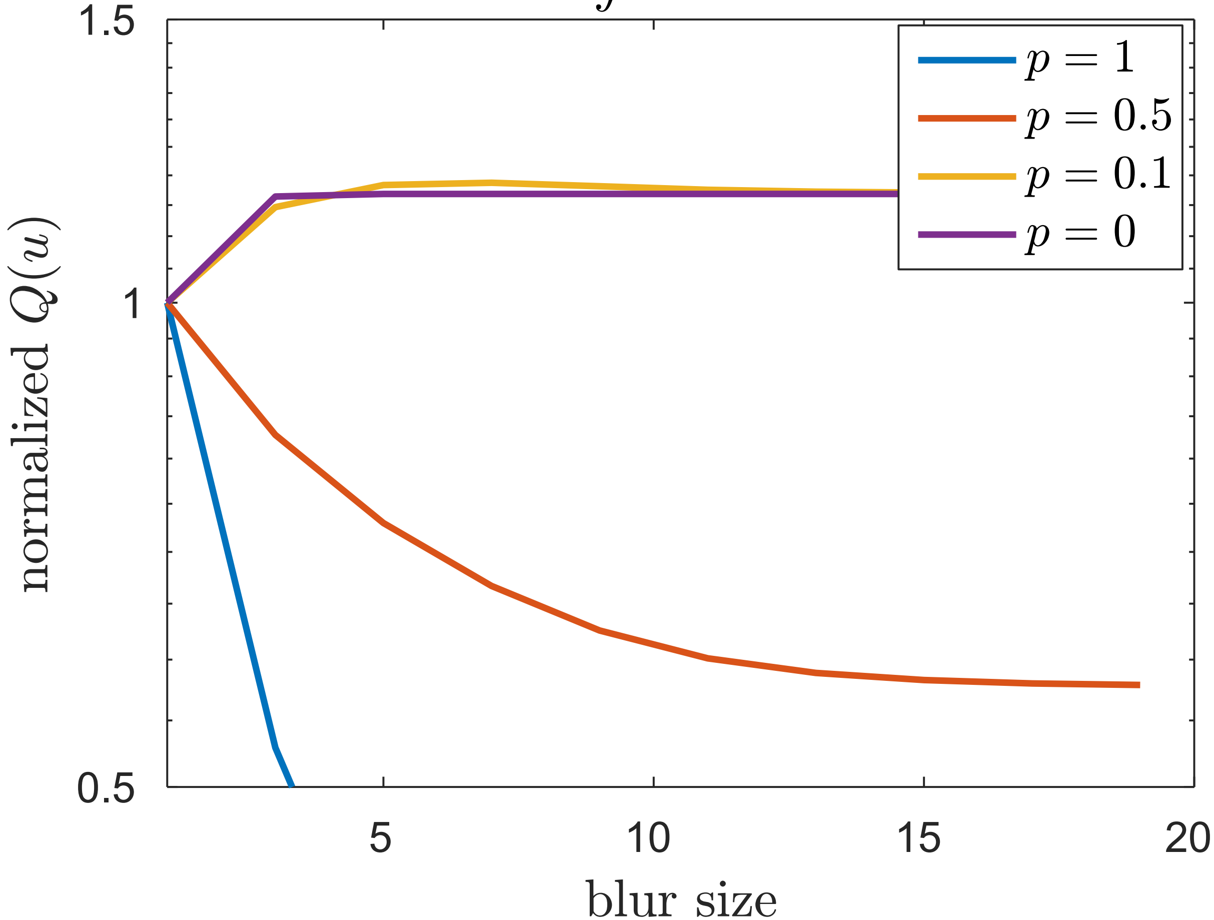


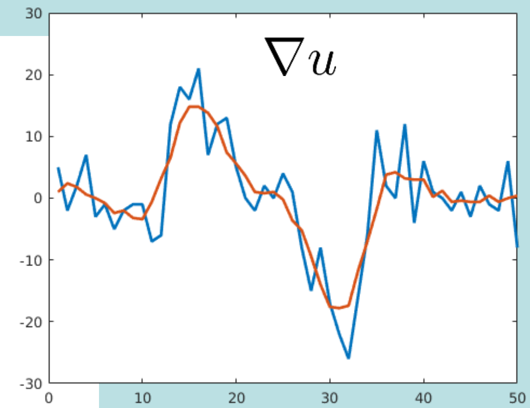
Artificially sparsify images



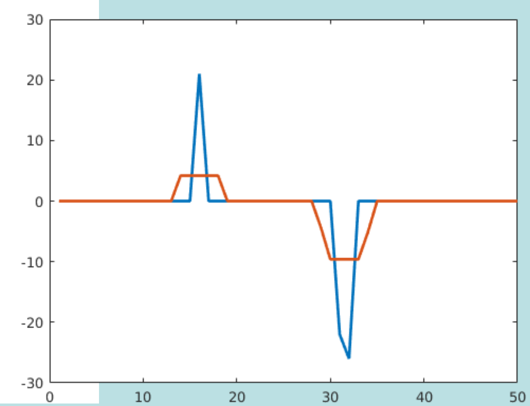
Ragularization favors blur

$$Q(u) = \int |\nabla u|^p$$





Artificially sparsify images

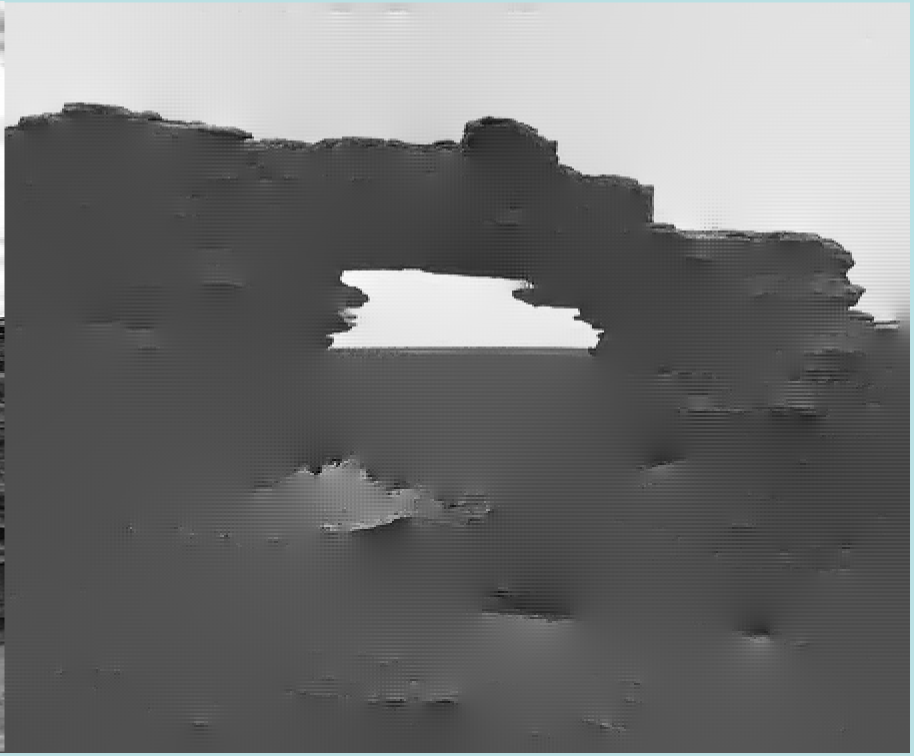


We need tricks

- To avoid “no-blur” solution:
  - Artificially sparsify image
    - Removing spikes
    - Sharpening
    - Adjusting priors on the fly
  - Hierarchical approach
  - Learn image prior with CNN

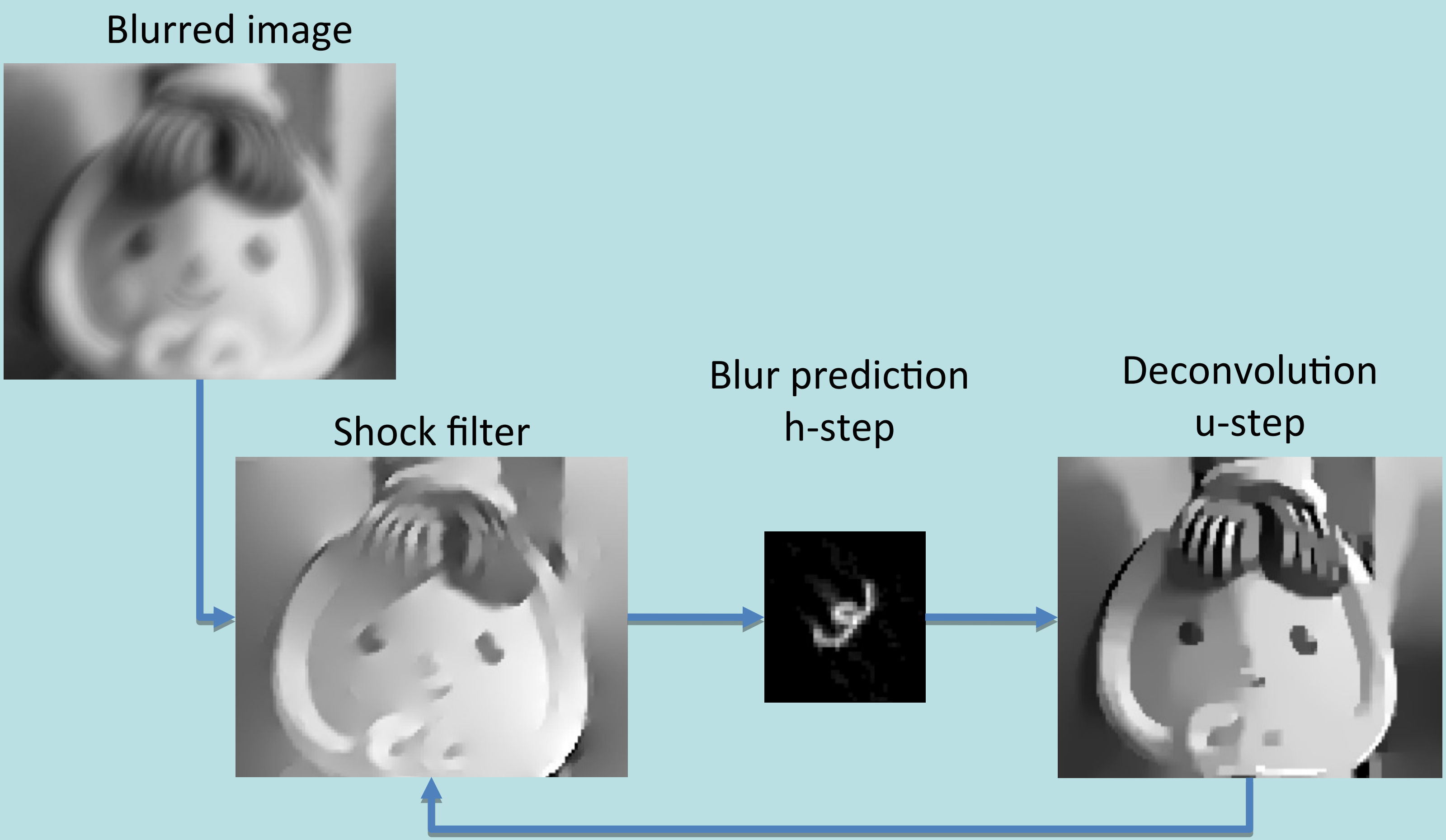
Chan TIP1998
Shan SigGraph08
Cho SigGraph 09
Xu ECCV09, 13
Almeida TIP10
Krishnan 11
Zhong 13
Sun 13
Michael 14
Perrone 15
Pan 16
Li CVPR18

Removing spiky objects



Reconstructed image with small objects removed

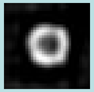
Artificial sharpening



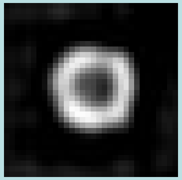
Hierarchical deconvolution

Scale N-2

Scale N-1



Scale N



image

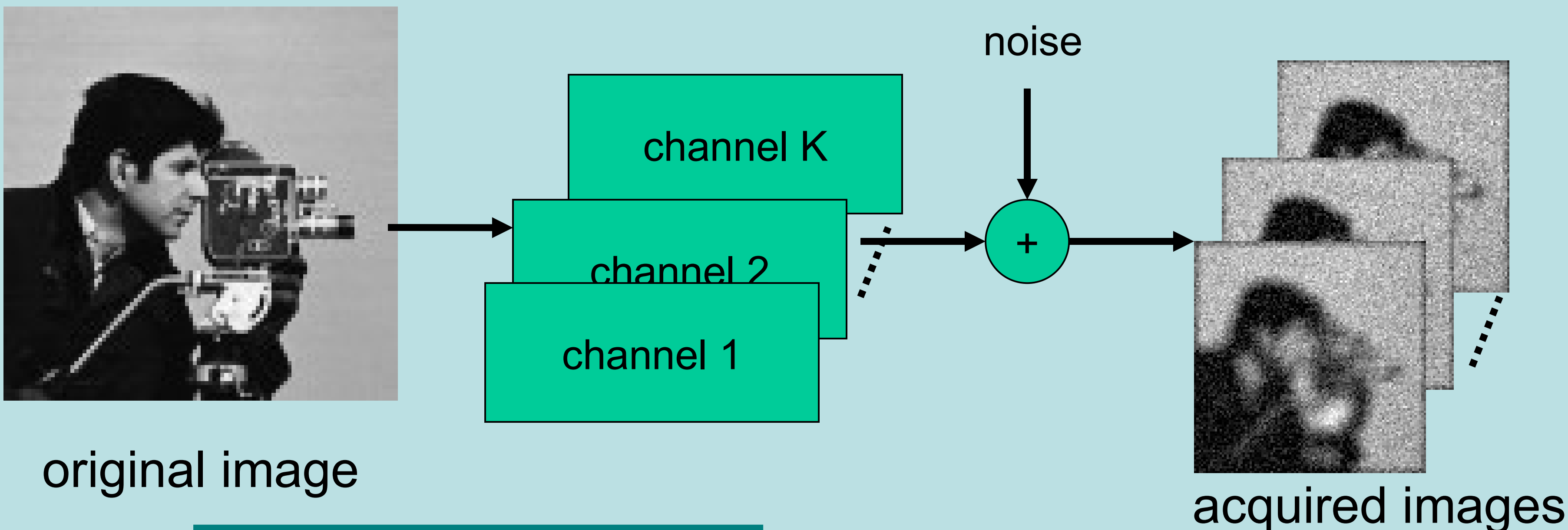
blur

Example of VB blind deconvolution

Blurred image *z(x)*

Reconstructed image *u(x)*

Multi-Channel Acquisition Model



$$[u * h_k](x) + n_k(x) = z_k(x)$$

Blind Deconvolution

- Acquisition model

$$z_k = (h_k * u) + n_k$$

- Minimization problem

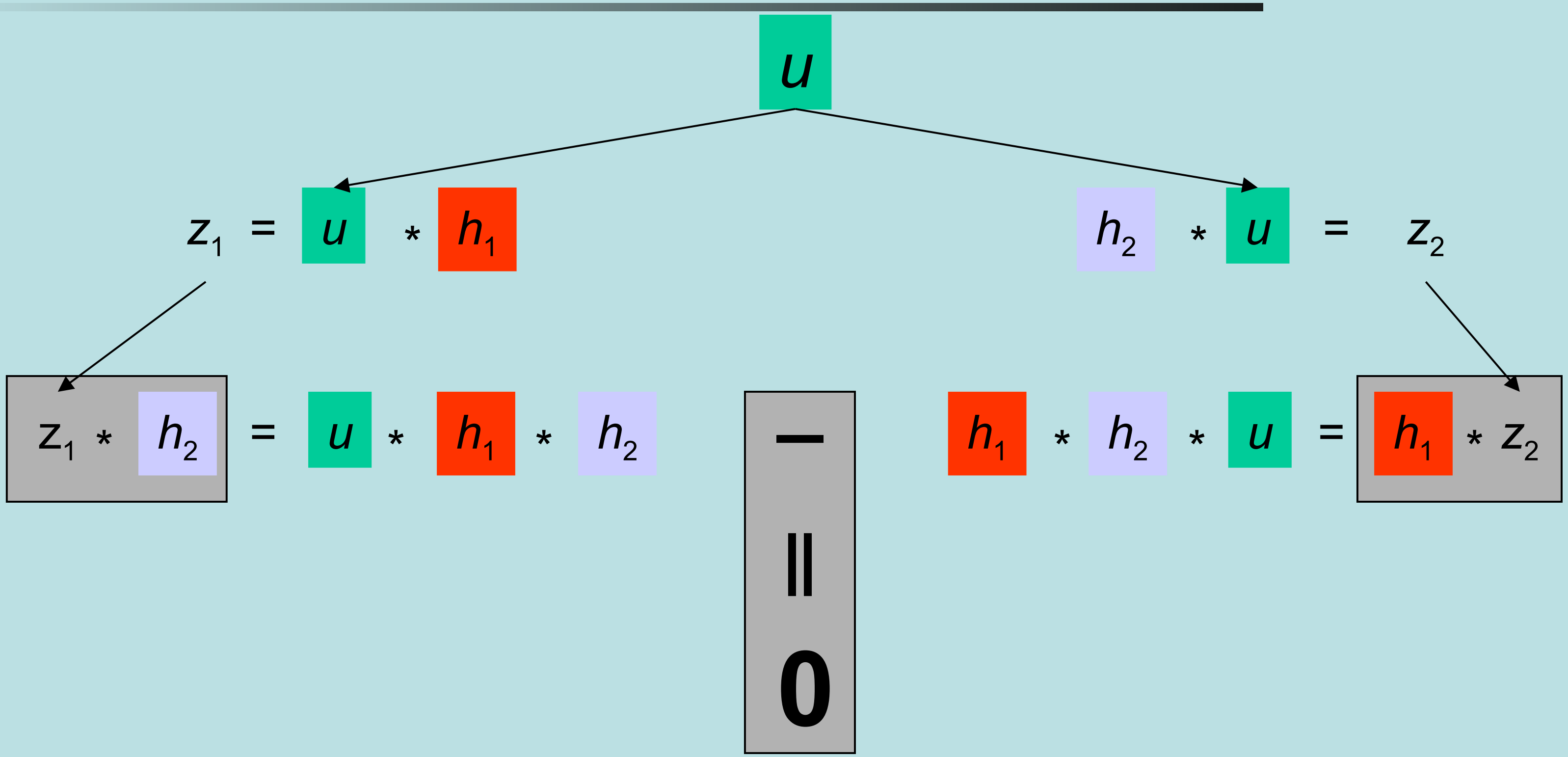
$$F(u, \{h_k\}) = \frac{1}{2} \sum_{k=1}^{K} \int_{\Omega} |z_k - h_k * u|^2 dx + \lambda \int_{\Omega} \phi(|\nabla u|) dx + \gamma R(\{h_k\})$$

Data term

Image regularization term

Blur Regularization term

Blur Regularization Term



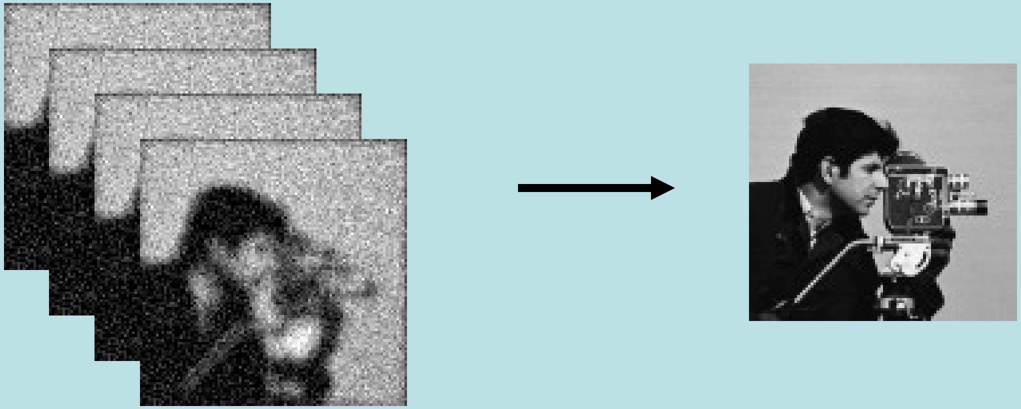
$$R(\{h_i\}) = \frac{1}{2} \sum_{1 \le i,j \le K} \| z_i * h_j - z_j * h_i \|^2$$

Alternating Minimization

Minimization of $F(u,\{h_k\})$ over u and h_k alternates.

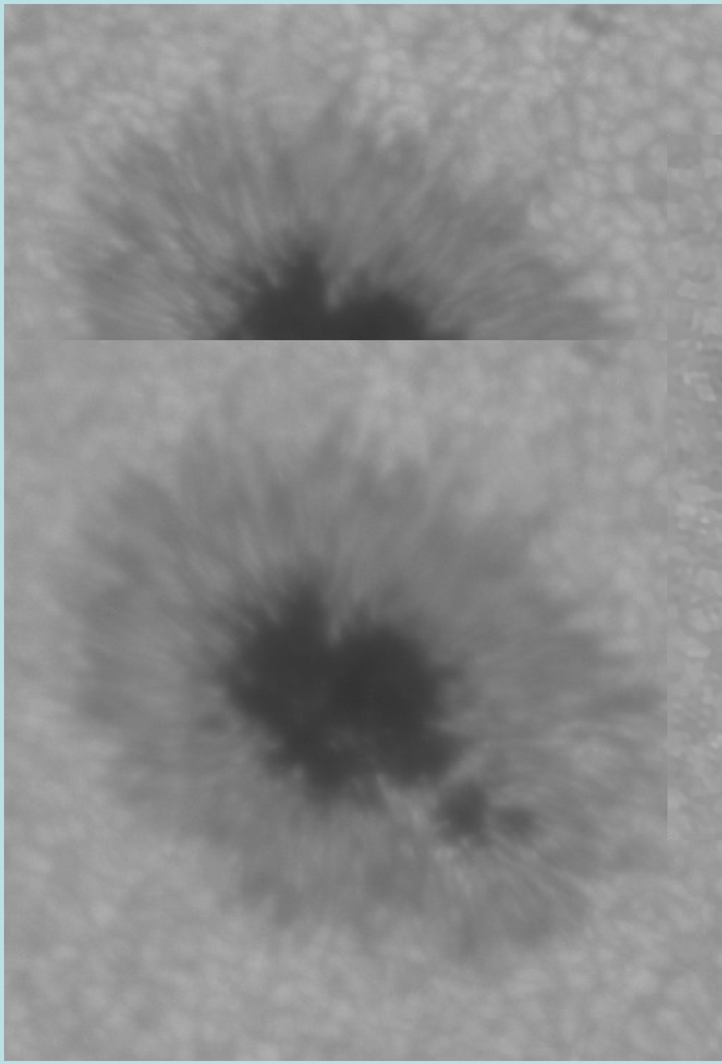
Input: Blurred images and estimation of the blur size

Output: Reconstructed image and the blurs

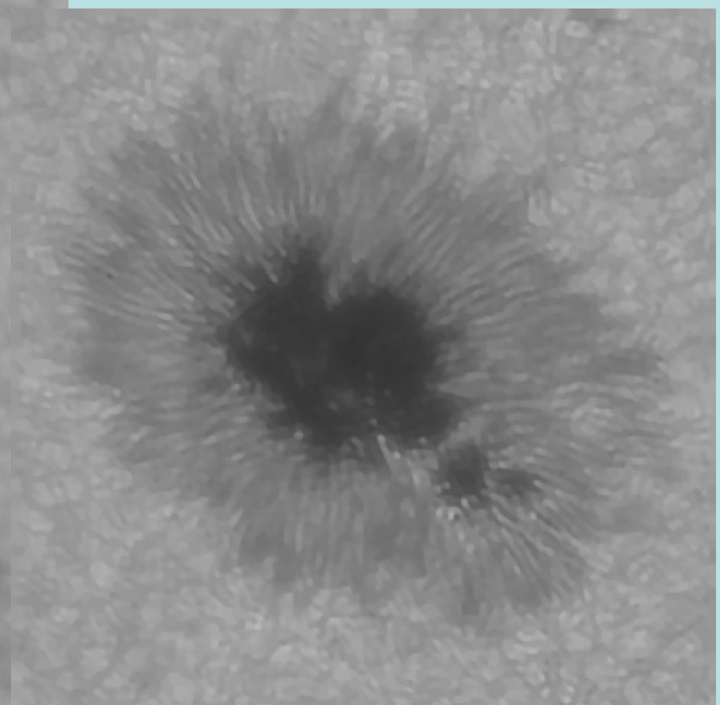


Astronomical Imaging

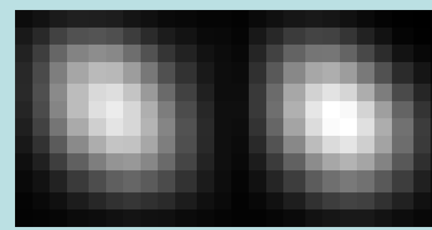
Degraded images



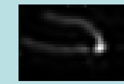
Reconstructed image

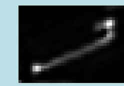


Blur estimation




Rio de Janeiro
Brasil
Centrum Chodov
05.04.08
Ján Kuva



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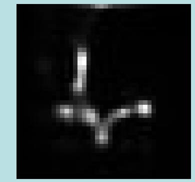
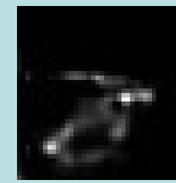
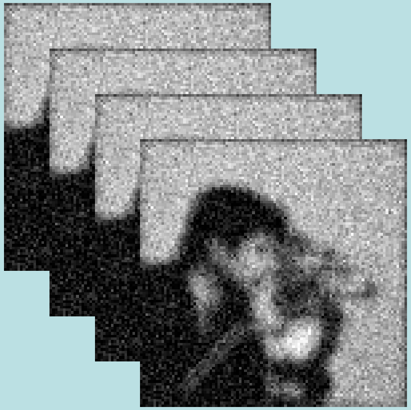
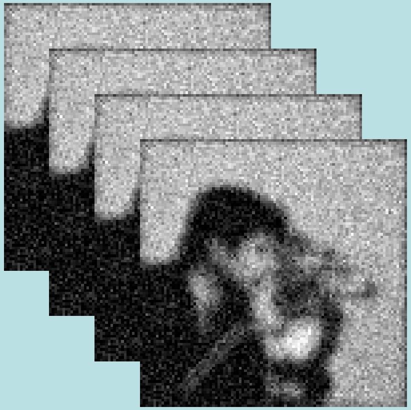




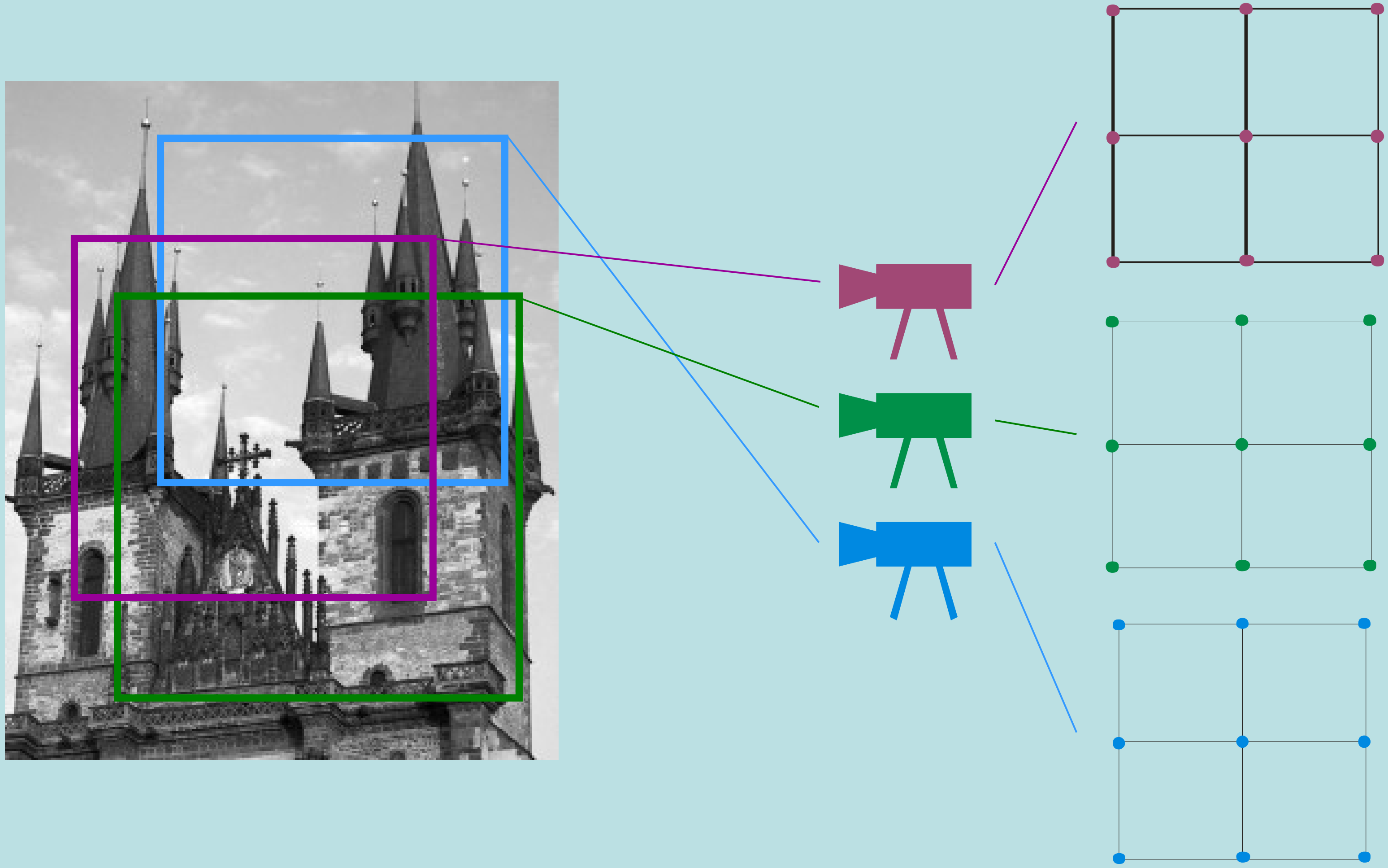
Multichannel Deconvolution



Super-resolution

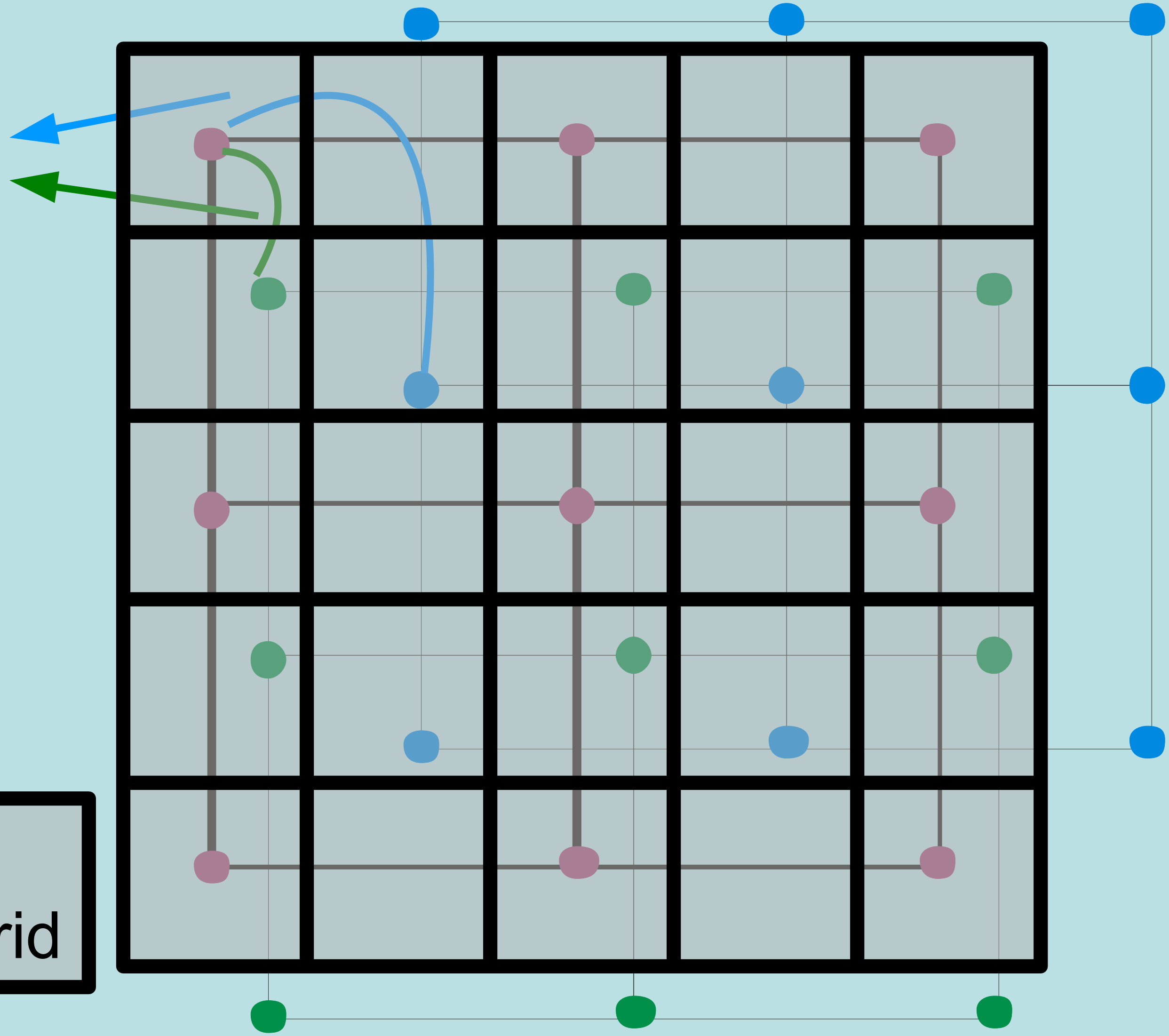


Super-resolution



Super-resolution

Sub-pixel shifts



Interpolation on
a high-resolution grid

Superresolution

- Acquisition model

$$z_k = D(h_k * u) + n_k$$

- Minimization problem

$$F(u, \{h_k\}) = \frac{1}{2} \sum_{k=1}^{K} \int_{\Omega} |z_k - D(h_k * u)|^2 dx + \lambda \int_{\Omega} \phi(|\nabla u|) dx + \gamma R(\{h_k\})$$

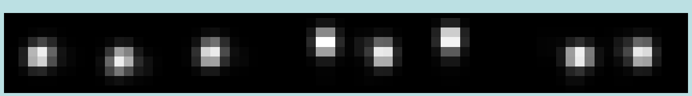
Data term

Image regularization term

Blur Regularization term

Superresolution

rough registration



Superresolved image (2x)

Optical zoom (ground truth)

SR limits

original

8 images

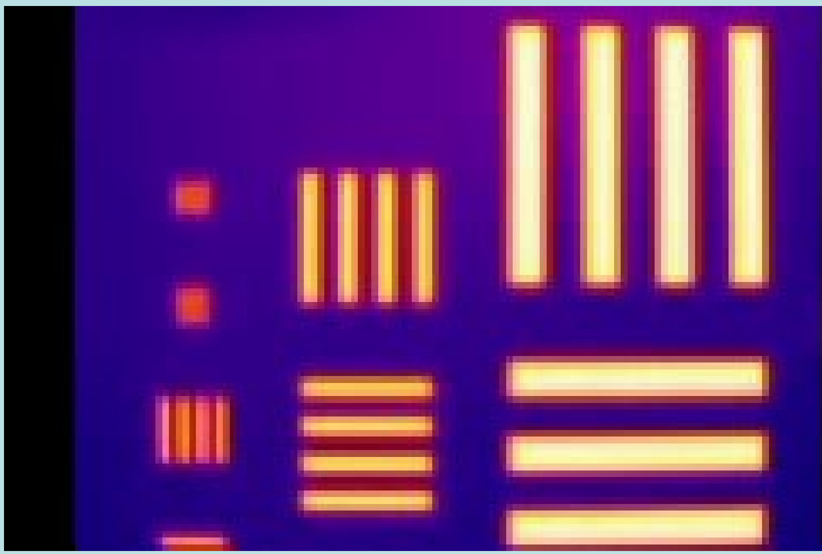
SR 2x

SR 3x

Superresolution of Video

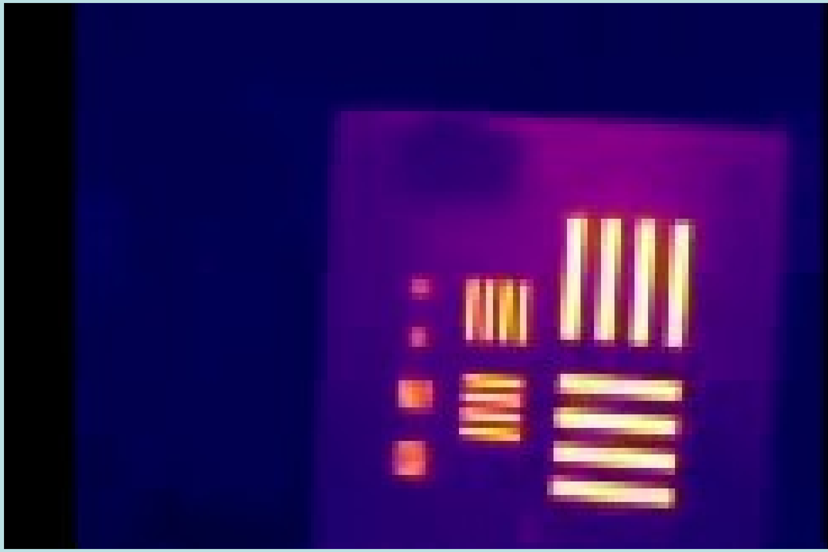
Interpolated video

Super-resolved video (2x)

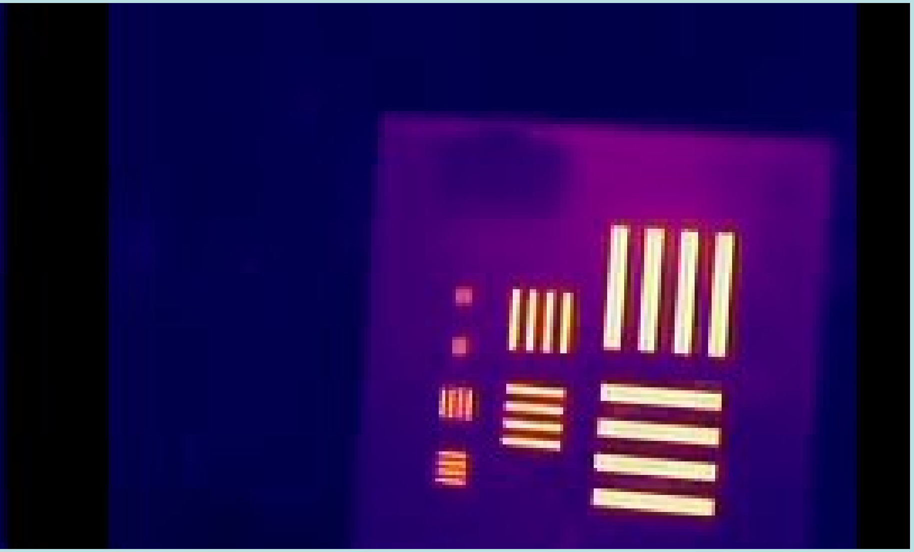


Interpolated video

Super-resolved video (2x)



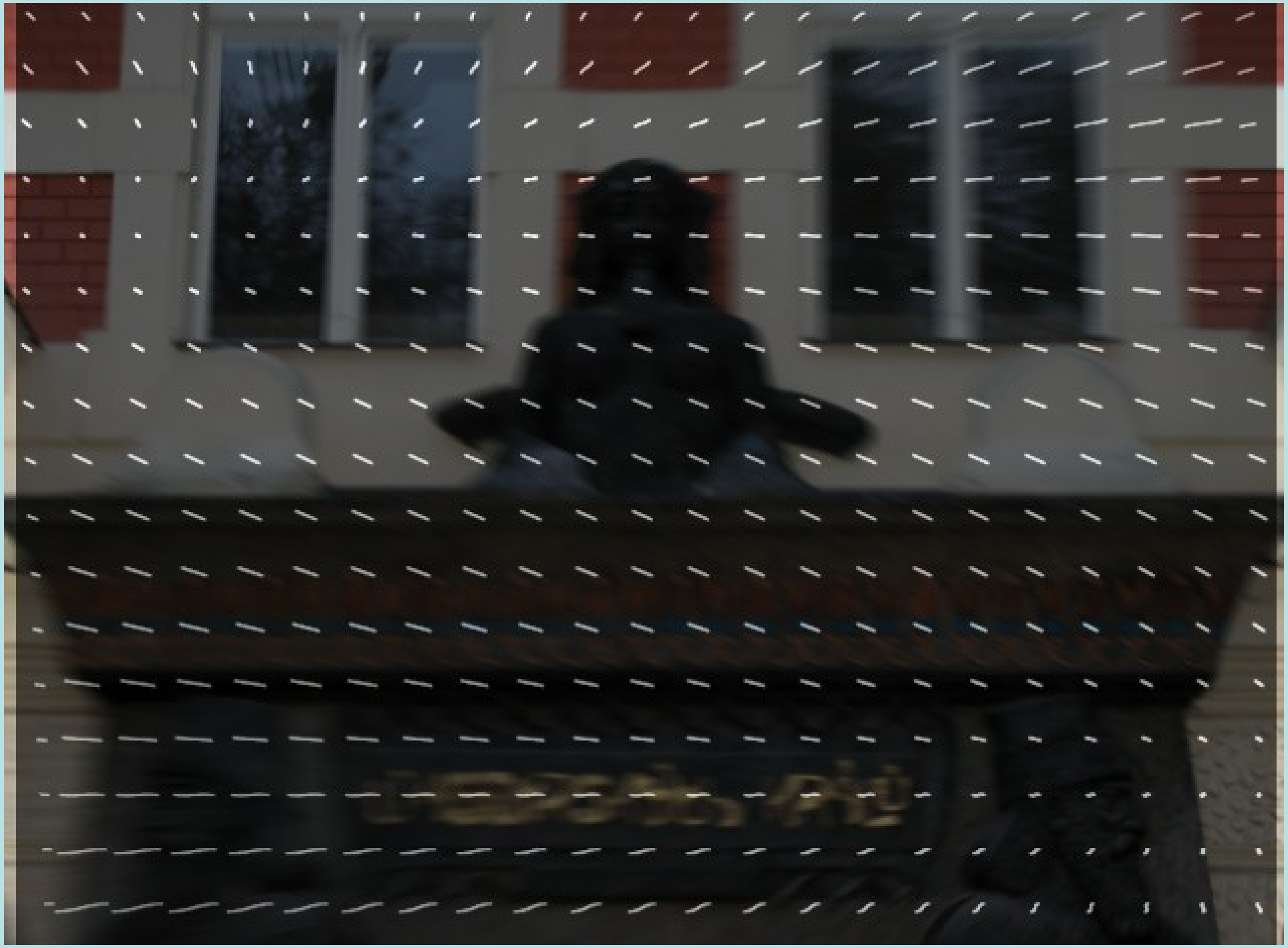
Interpolated video



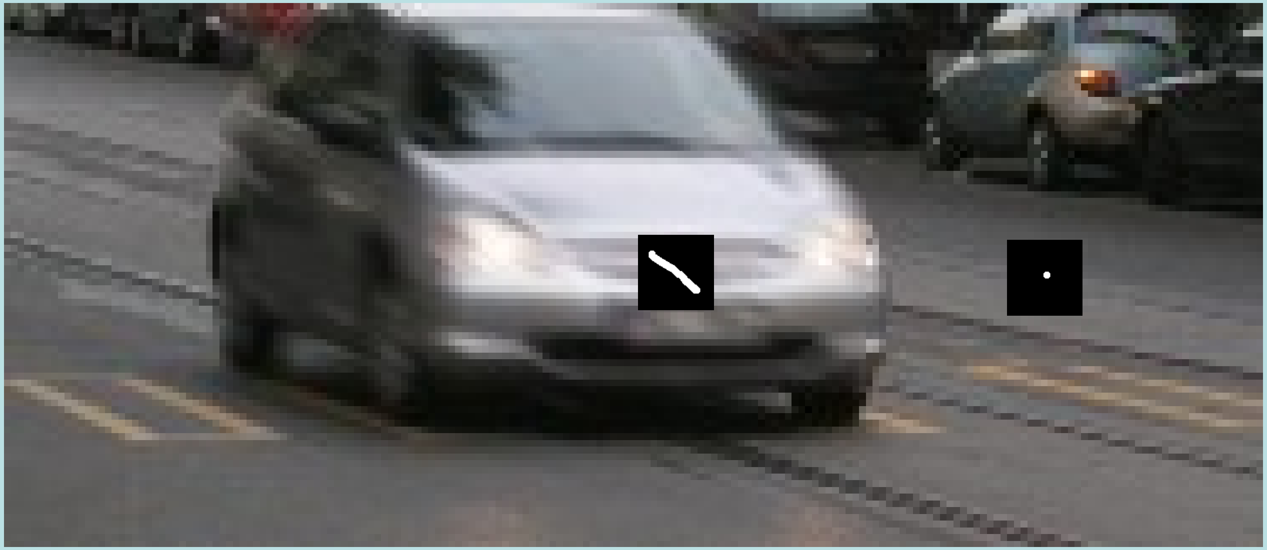
Super-resolved video (2x)

Space-variant blur

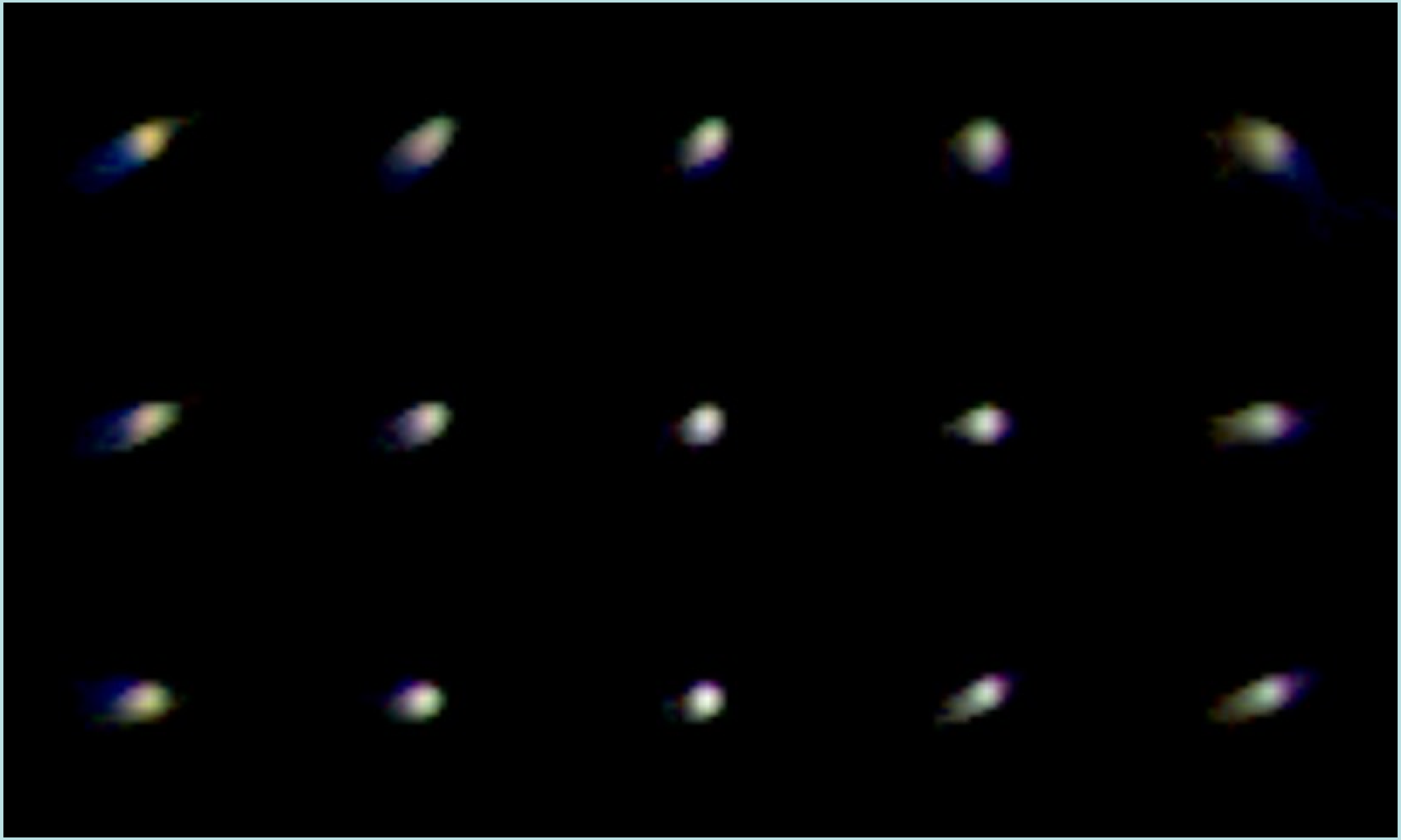
Camera Motion



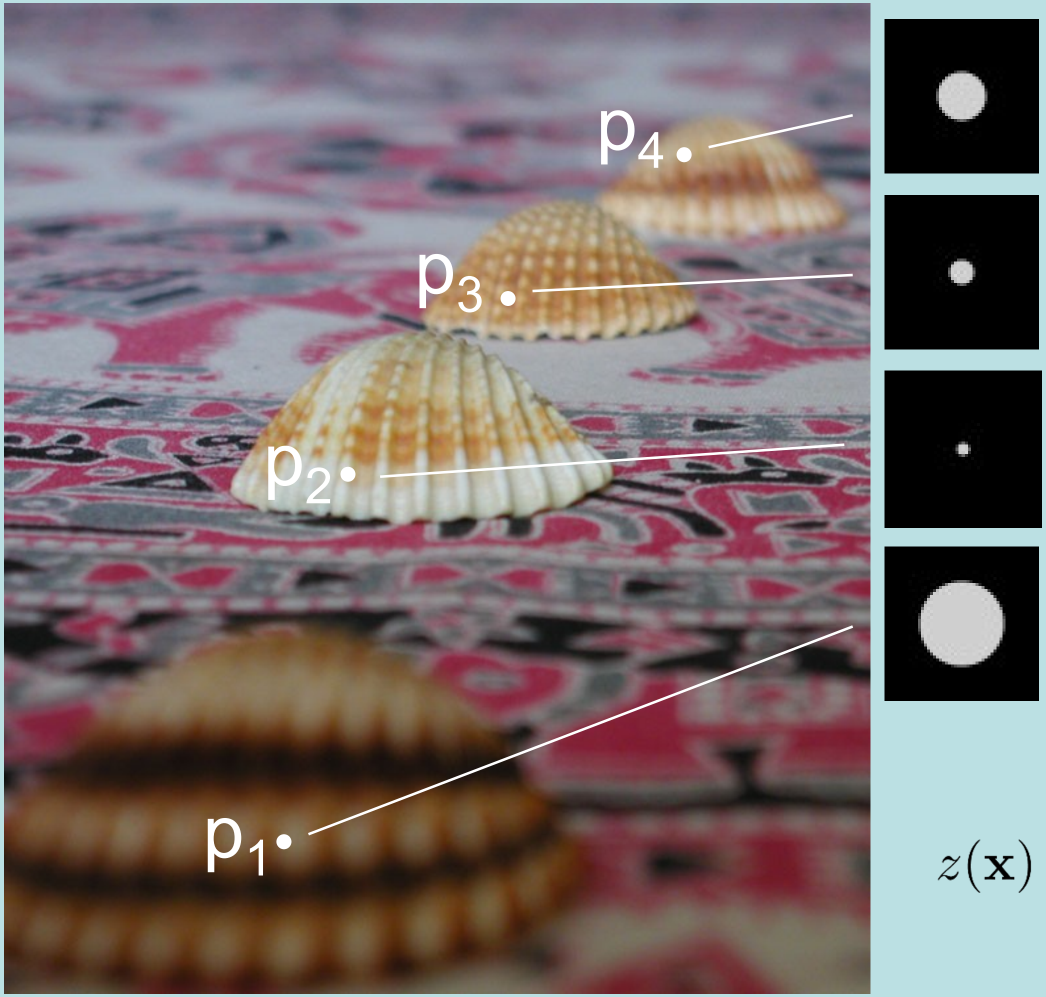
Object Motion



Optical Abberations



Space-variant Out-of-focus Blur



$h(\mathbf{x};\mathbf{p}_4)$

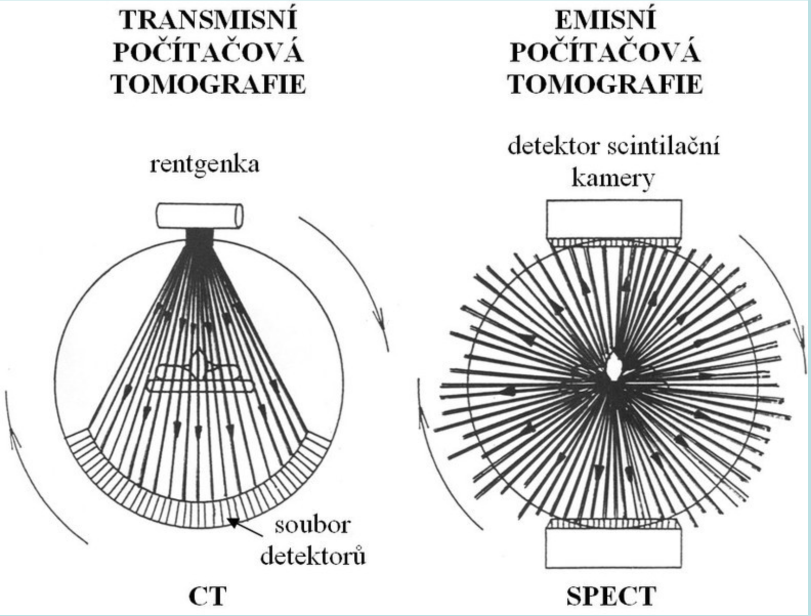
$\vdots$

$h(\mathbf{x};\mathbf{p}_1)$

$$z(\mathbf{x}) = \int u(\mathbf{x}-\mathbf{s})h(\mathbf{s};\mathbf{x}-\mathbf{s})d\mathbf{s}$$

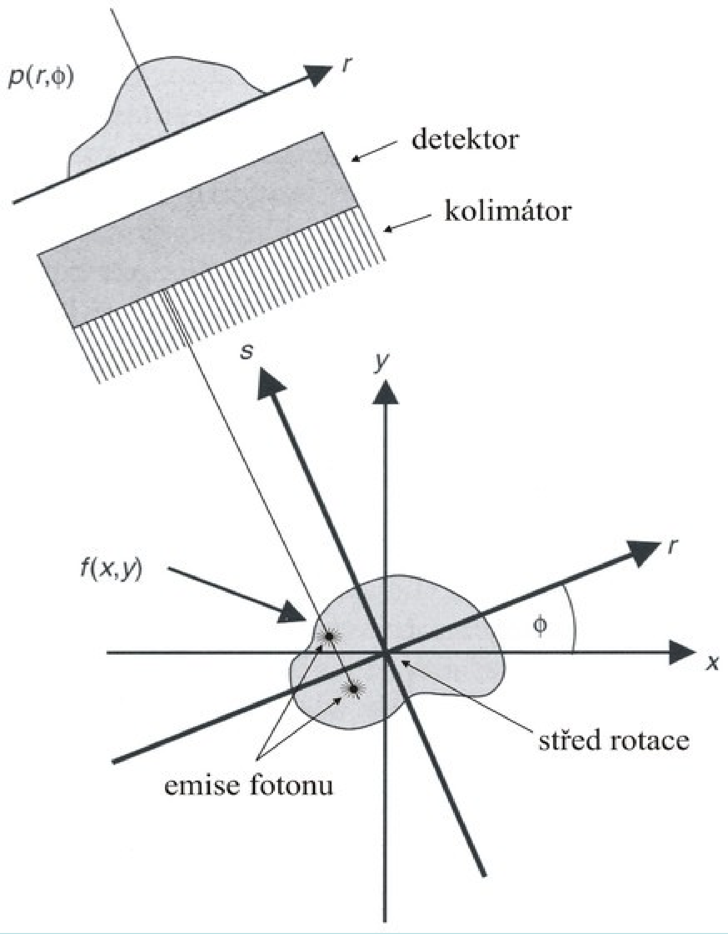
Tomographic Reconstruction

- CT X-rays
- SPECT gamma rays
- MRI electromagnetic waves
- PET positron-electron annihilation



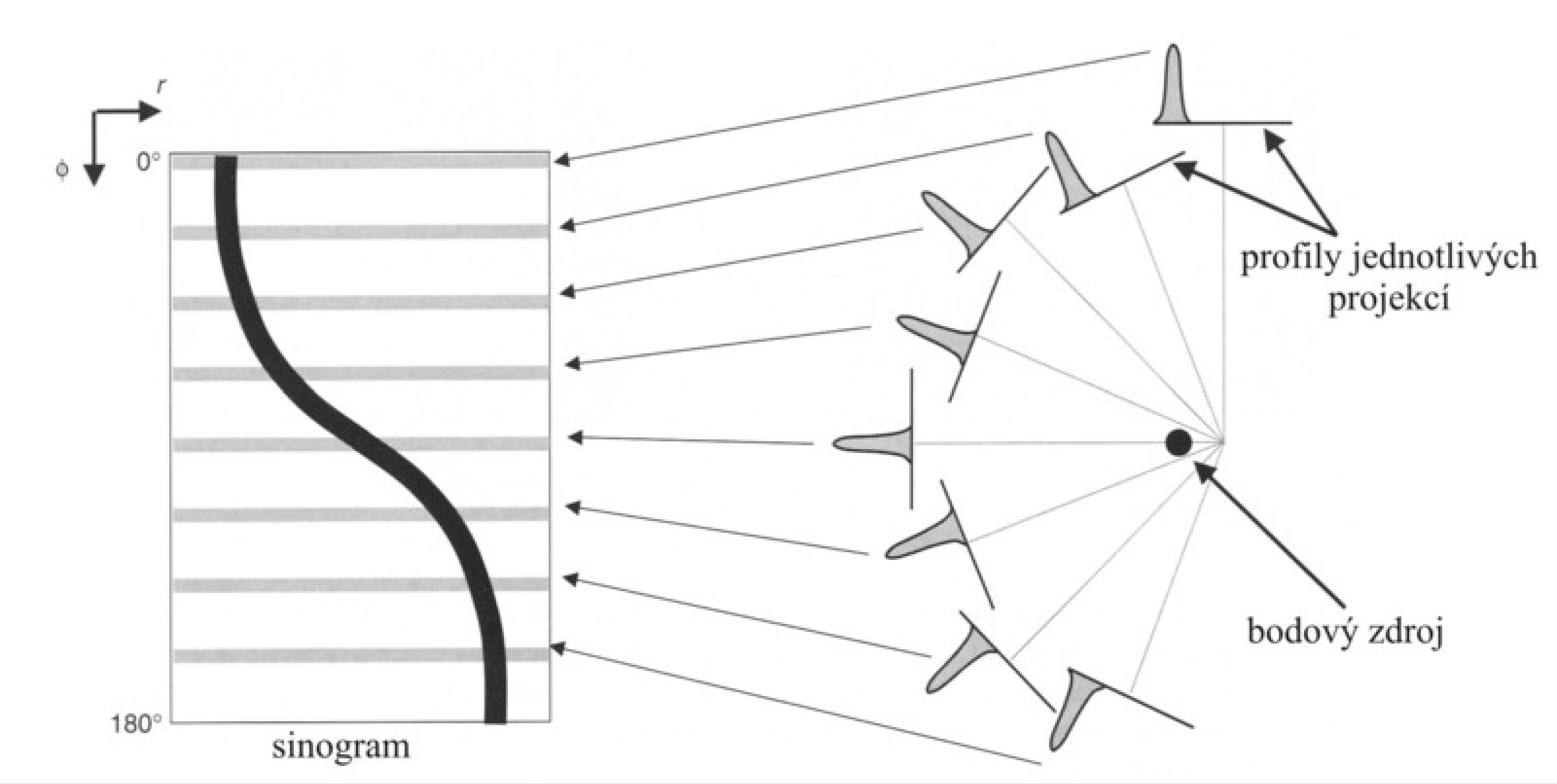
Tomography Principle

- 1D projections of 2D objects

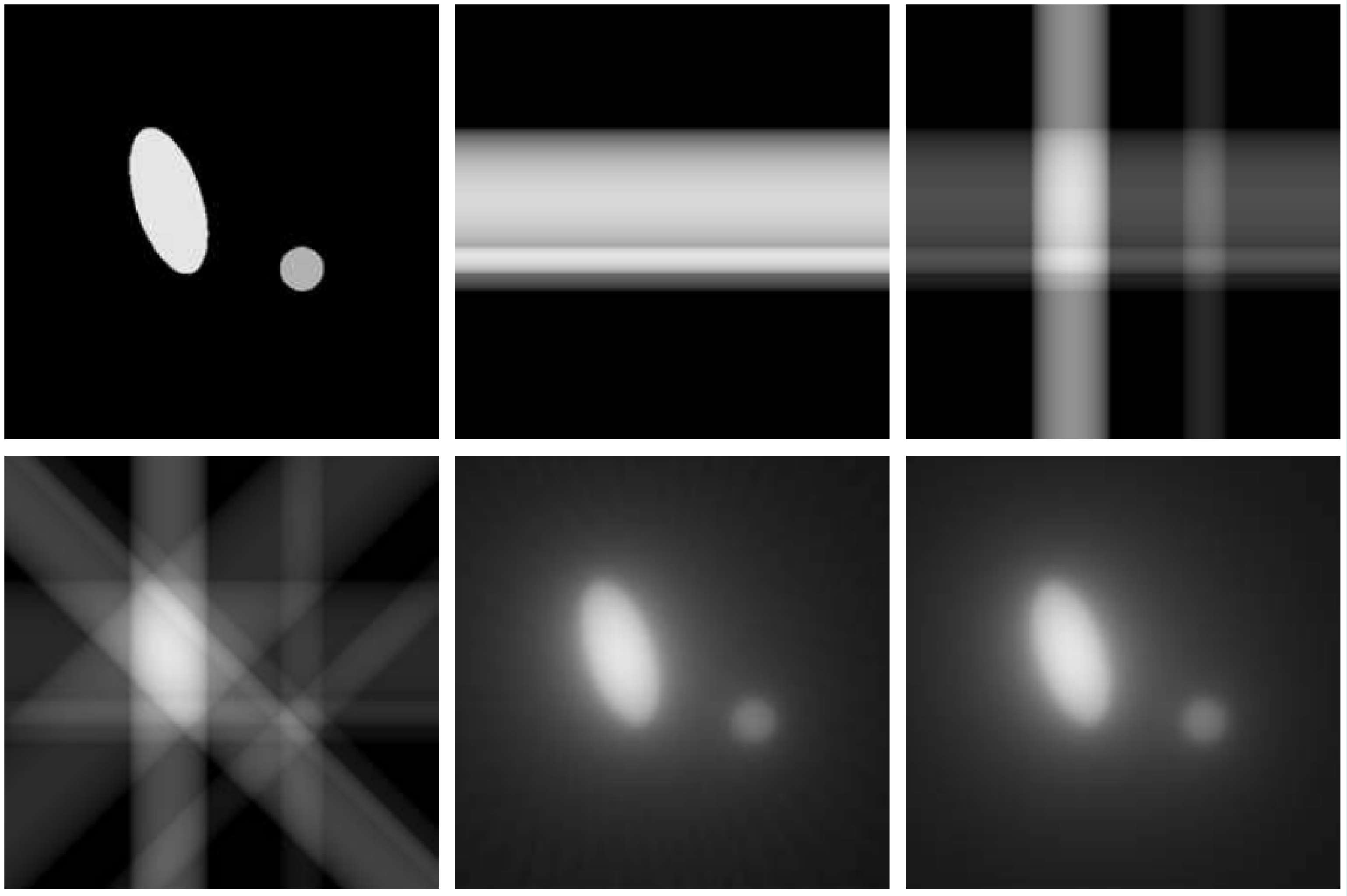


Sinogram

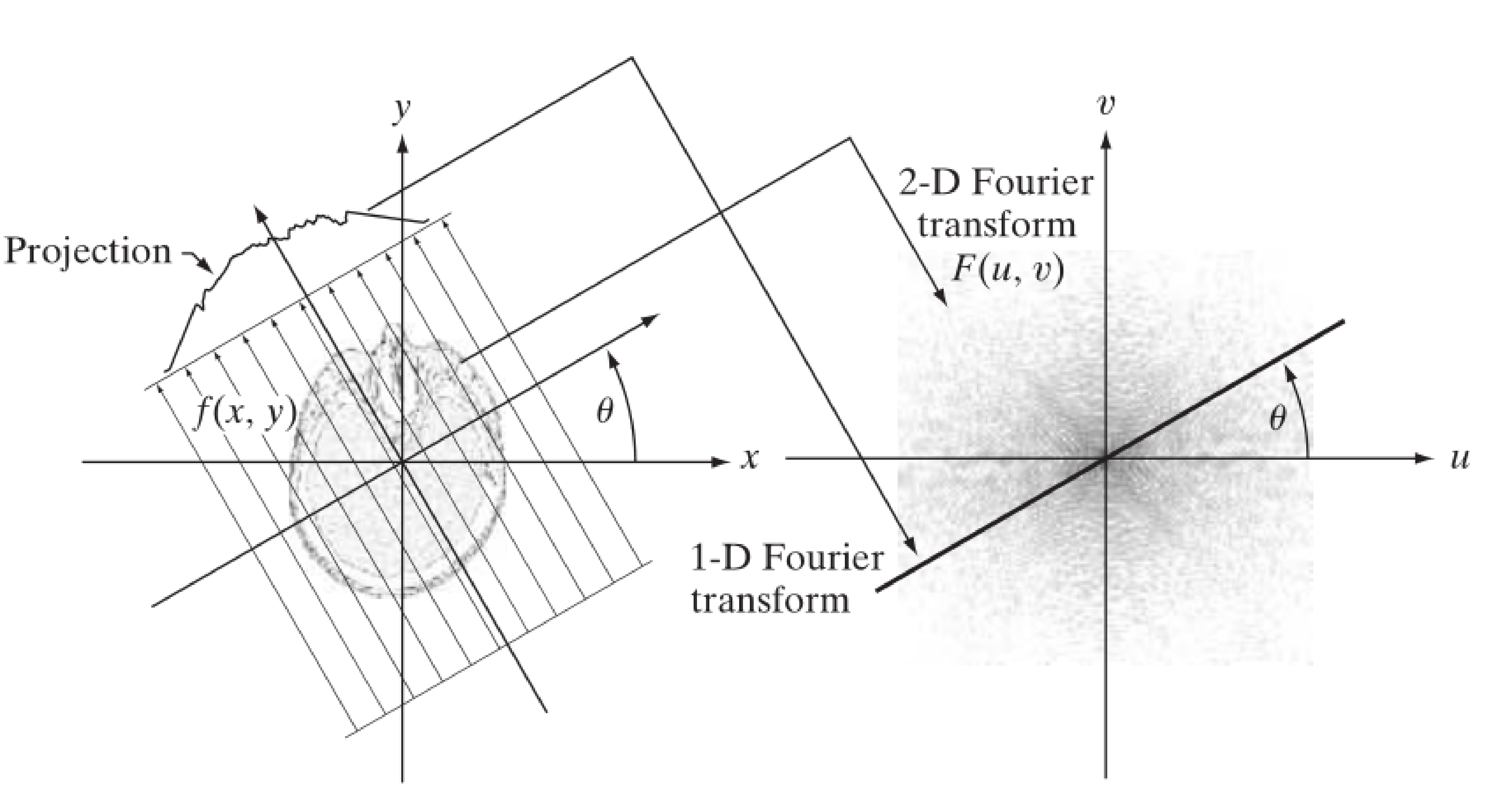
- Projections (sinogram) = Radon Transform
- Reconstruction → Inverse Radon (Filtered Back Projection)



Back Projection



Projection-Slice Theorem



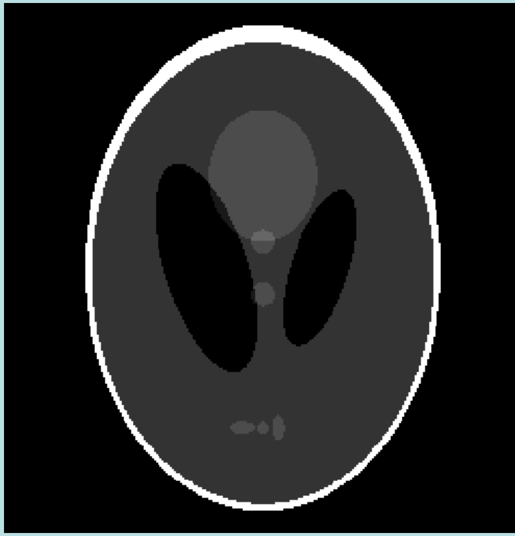
Variational Reconstruction

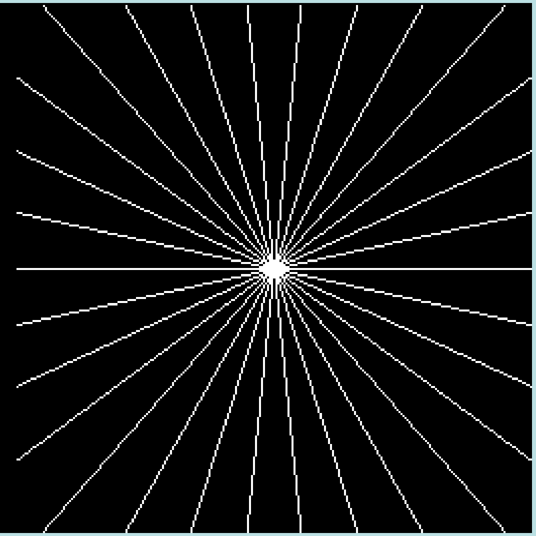
- R … operator performing projections
- z … sinogram

- Our optimization problem is

$$F(u) = \frac{1}{2}\int_{\Omega} |z - Ru|^2 dx + \lambda \int_{\Omega} \phi(|\nabla u|) dx$$

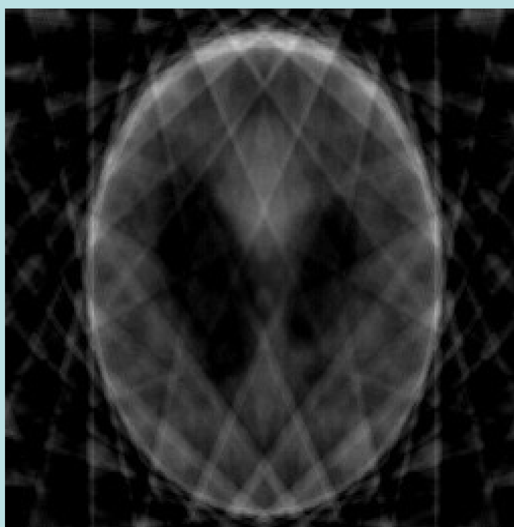
original





15 projections
(in Fourier domain)

Back Projection



Variational
Reconstruction

End