Optical Flow

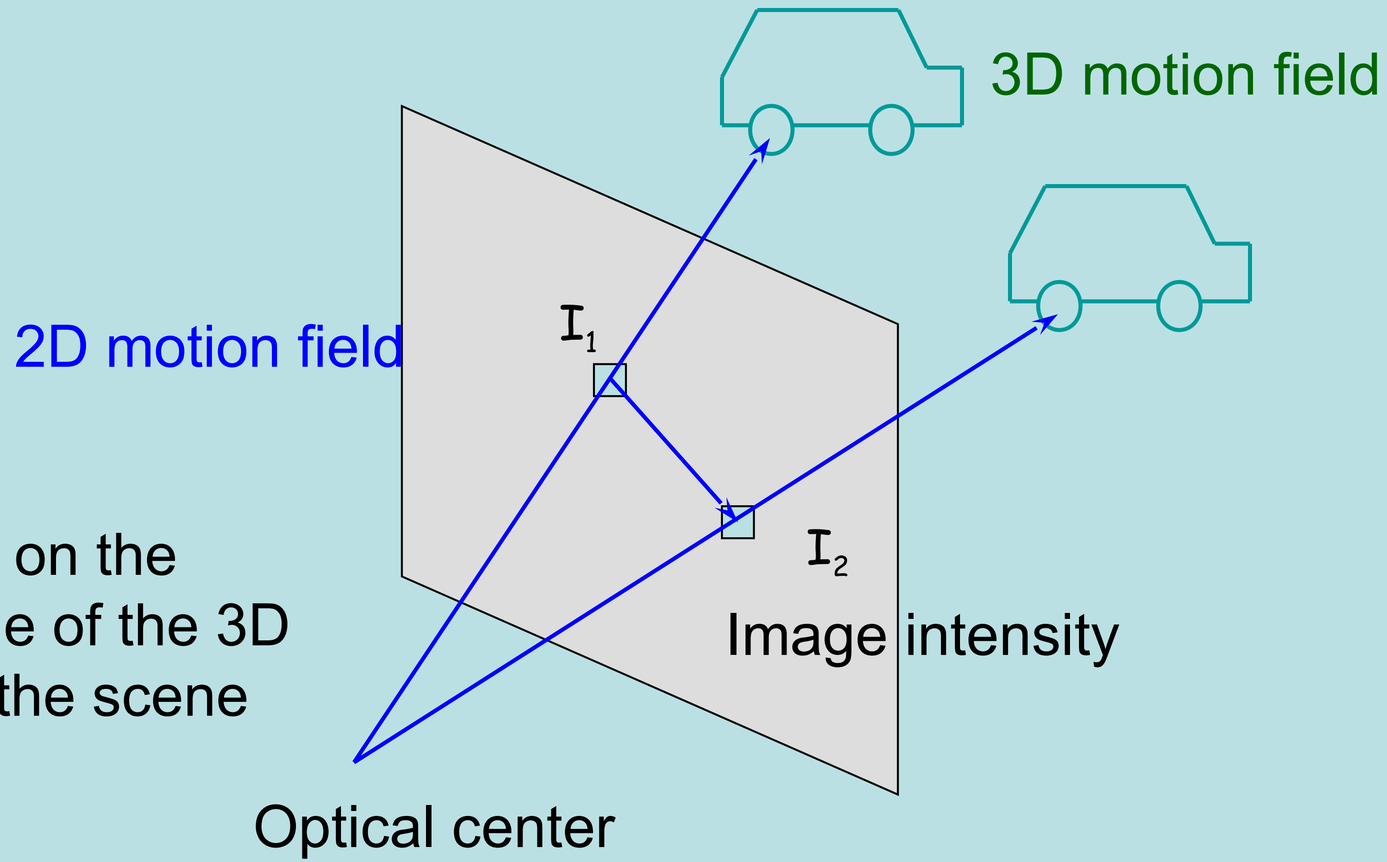
Motion Detection

Image Sequence

Sequence of images contains information about the scene,

We want to estimate motion (using variational formulation)

2D Motion Field



Projection on the image plane of the 3D velocity of the scene

Homography

- projective transform
- Pinhole camera
  - Rotating camera and arbitrary 3D scene
  - Arbitrarily moving camera and planar scene

$$x' = \frac{h_1 x + h_2 y + h_3}{h_7 x + h_8 y + h_9}$$

$$y' = \frac{h_4 x + h_5 y + h_6}{h_7 x + h_8 y + h_9}$$

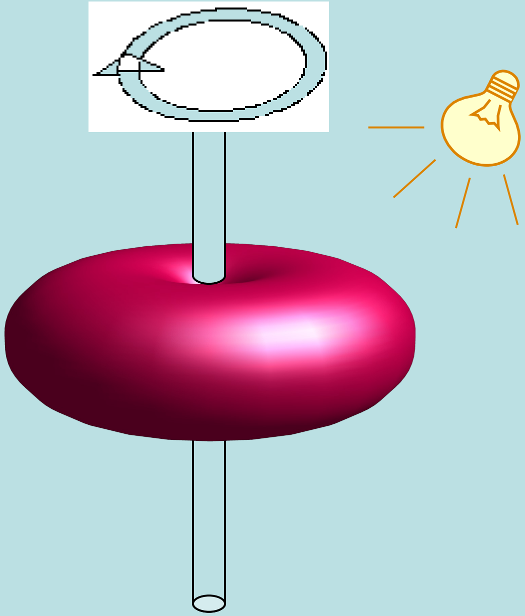
$$\begin{bmatrix} x'd \\ y'd \\ d \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

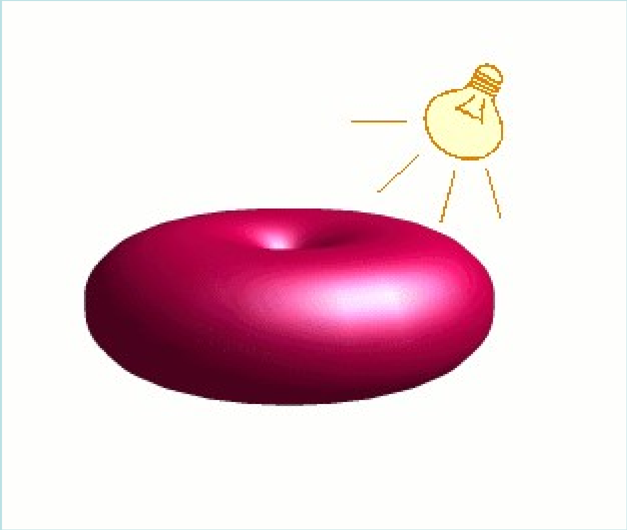
Optical Flow

What we are able to perceive is just an apparent motion, called

Optical Flow

(motion, observable only through intensity variations)





Intensity remains constant – no motion is perceived

No object motion, moving light source produces intensity variations

Brightness Constancy

- Intensity of a point keeps constant along its trajectory (reasonable for small displacements)
- $I(t,\mathbf{x})$ … intensity of the pixel $\mathbf{x}=(x_1,x_2)$ at t
- Trajectory $(t,\mathbf{x}(t))$ starts at $\mathbf{x}_0 = \mathbf{x}(t_0)$

$$I(t, \mathbf{x}(t)) = I(t_0, \mathbf{x}_0) \quad \forall t$$

- Differentiate with respect to time

$$\frac{d\mathbf{x}}{dt} \cdot \nabla I + \frac{\partial I}{\partial t} = 0 \quad \text{at} \quad t = t_0$$

- Optical flow as the velocity field $\mathbf{v}(t_0) = \frac{d\mathbf{x}}{dt}(t_0)$

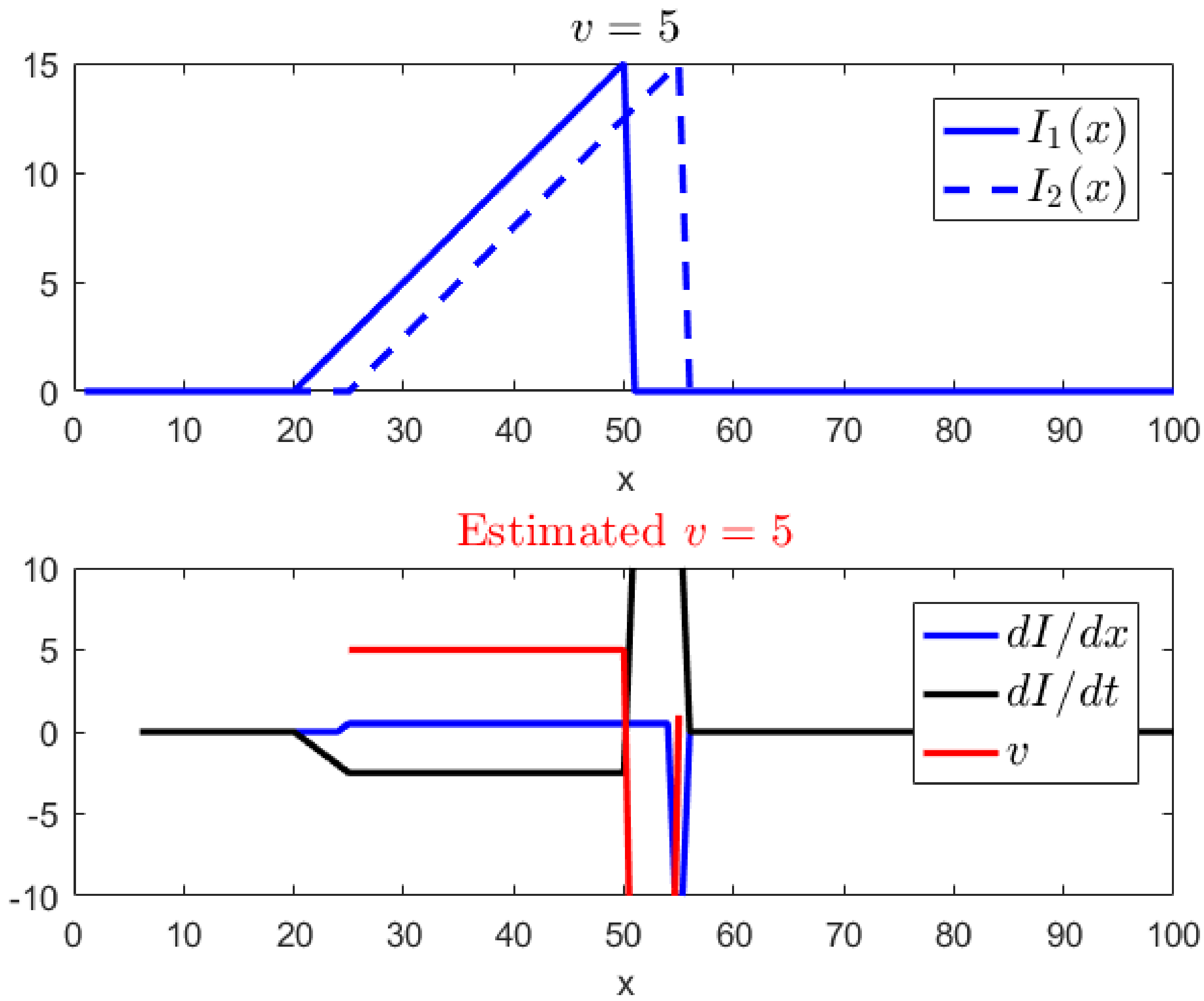
Discrete version

- Two input images: $I_1(\mathbf{x}) = I(t_1, \mathbf{x})$
$$I_2(\mathbf{x}) = I(t_2, \mathbf{x})$$

- Taylor:

$$I_1(\mathbf{x}) = I_2(\mathbf{x} + \mathbf{v}) = I_2(\mathbf{x}) + \mathbf{v} \cdot \nabla I_2(\mathbf{x})$$

$$0 = I_2(\mathbf{x}) - I_1(\mathbf{x}) + \mathbf{v} \cdot \nabla I_2(\mathbf{x})$$



$v = 5$
15
10
5
0
0
10
20
30
40
50
60
70
80
90
100
x
$I_1(x)$
$I_2(x)$
Estimated $v = 5$
10
5
0
-5
-10
dI/dx
dI/dt
v
x

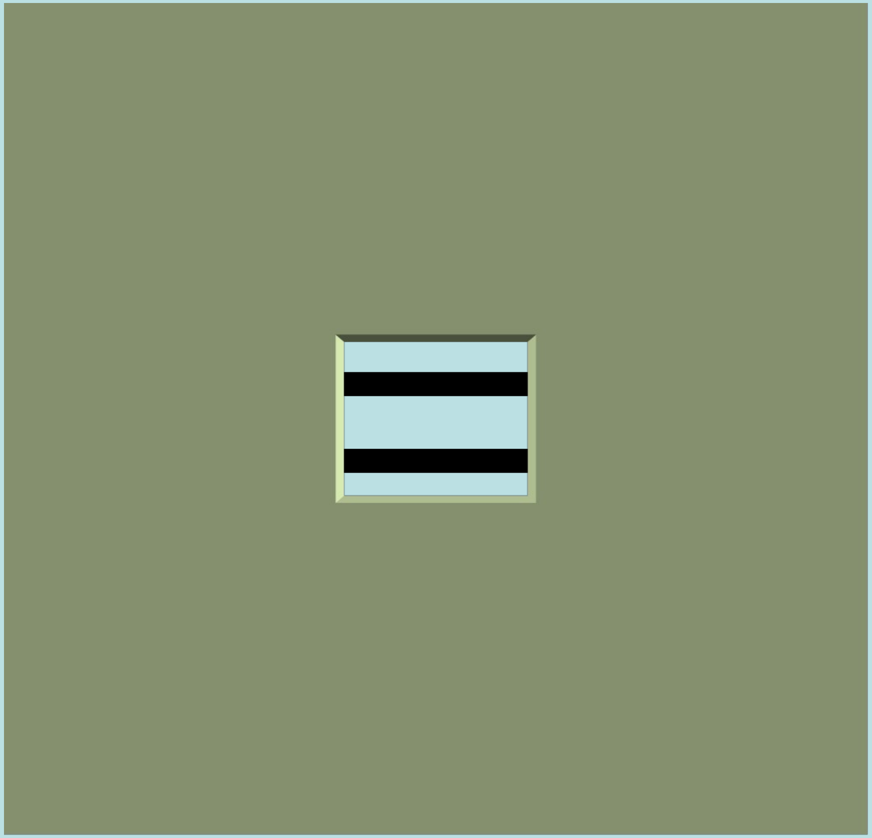
Optical Flow Constraint

- Given: sequence $I(t,\mathbf{x})$
- Find: velocity $\mathbf{v}(\mathbf{x})=[v_1(\mathbf{x}),v_2(\mathbf{x})]$ such that

$$\mathbf{v}(\mathbf{x}) \cdot \nabla I(t, \mathbf{x}) + I_t(t, \mathbf{x}) = 0$$

- Velocity field has 2 components but we have one scalar equation => ???

Aperture problem



Solving Aperture Problem

- Second order derivative constraint
- Least-square fit (constant in spatial, temporal or spectral domain)
- Regularization

Second order constraint

- Conservation of the image gradient along the trajectory $(t, \mathbf{x}(t))$

$$\frac{d\nabla I}{dt}(t, \mathbf{x}(t)) = 0$$

$$\begin{bmatrix} I_{x_1 x_1} & I_{x_2 x_1} \\ I_{x_1 x_2} & I_{x_2 x_2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} I_{x1t} \\ I_{x2t} \end{bmatrix} = 0$$

- No rotation and/or dilation, sensitive to noise

Least-square fit

- Velocities constant in small window w

$$\min_{\mathbf{v}} F(\mathbf{v}) = \min_{\mathbf{v}} \int w^2(x - x_0)(\mathbf{v}(x_0) \cdot \nabla I + I_t)^2 dx$$

Weighted window

OFC

  - Too local, no global regularity
- Parametric velocity model

$$\min_{\mathbf{v}} F(\mathbf{v}) = \min_{a,b,c,d,e,f} F(\mathbf{v}) \qquad \mathbf{v}(x) = \begin{bmatrix} a + bx_1 + cx_2 \\ d + ex_1 + fx_2 \end{bmatrix}$$

  - Restrictive but e.g. homography is common

Parametric velocity model

- Homography $$\begin{bmatrix} v_1 d \\ v_2 d \\ d \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

- Find parameters $\mathbf{h} = [h_1, \ldots, h_9]$

  – OF: $I_{x_1} v_1 + I_{x_2} v_2 + I_t = 0$

$$I_{x_1} v_1 d + I_{x_2} v_2 d + I_t d = 0$$

Parametric velocity model

$$\begin{bmatrix} v_1 d \\ v_2 d \\ d \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

$$I_{x_1} v_1 d + I_{x_2} v_2 d + I_t d = 0$$

$$\left[\begin{array}{ccc|ccc|ccc} I_{x_1} x_1 & I_{x_1} x_2 & I_{x_1} & I_{x_2} x_1 & I_{x_2} x_2 & I_{x_2} & I_t x_1 & I_t x_2 & I_t \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}\right] \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = 0$$

$$h_9 = 1$$

$$\tilde{\mathbf{M}}\tilde{\mathbf{h}} = \begin{bmatrix} -I_t \\ \vdots \end{bmatrix} = \mathbf{b}$$

LS fit => $\arg\min_{\mathbf{h}} \|\tilde{\mathbf{M}}\tilde{\mathbf{h}} - \mathbf{b}\|^2$

Regularizing the Velocity Field

- Minimization problem

$$F(\mathbf{v}) = \frac{1}{2}\int_\Omega (\nabla I \cdot \mathbf{v} + I_t)^2 dx + \lambda \sum_{i=1}^{2} \int \phi(|\nabla v_i|) dx$$

Data term

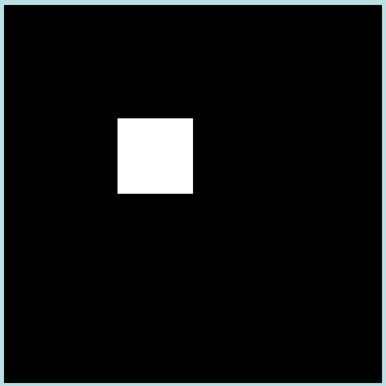
Weighting parameter

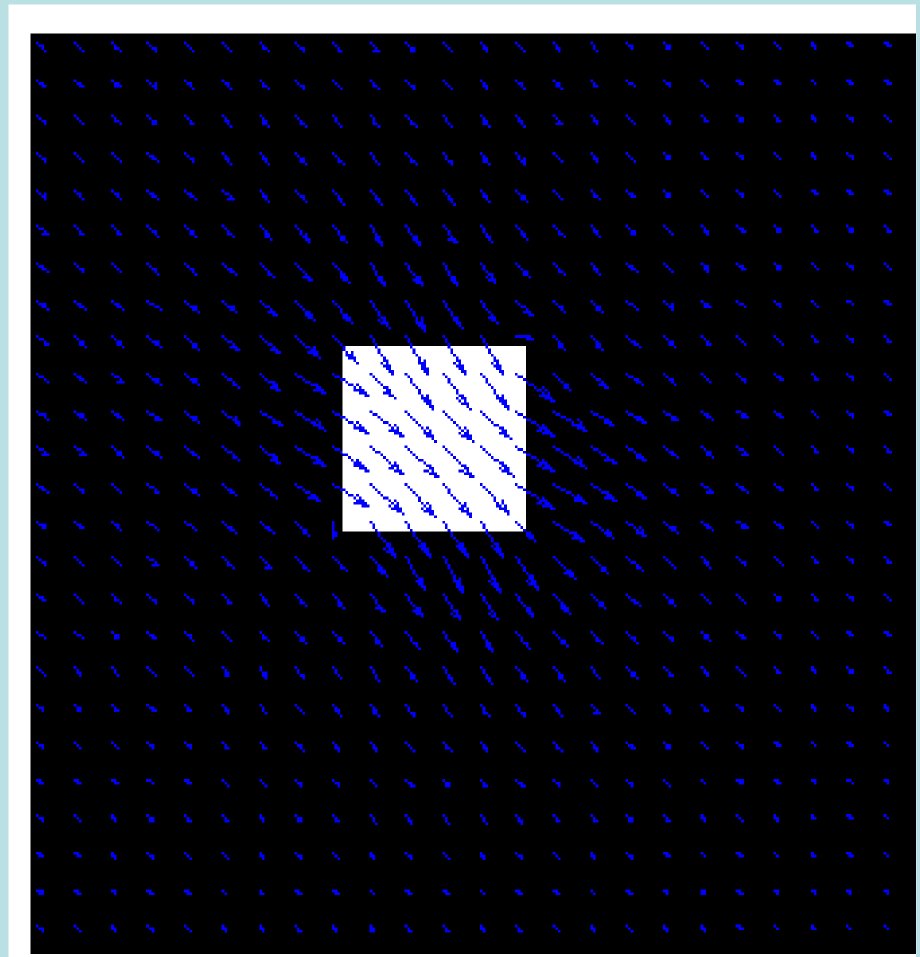
Regularization term

$$\phi(s) = s^2, \quad \phi(s) = \sqrt{s^2 + \epsilon}$$

Example

- Synthetic example





Homogeneous term

- No texture = no gradient → no way to estimate correctly the flow field
- So we force it to be zero

$$F(\mathbf{v}) = \frac{1}{2}\int_{\Omega}(\nabla I \cdot \mathbf{v} + I_t)^2 dx + \lambda \sum_{i=1}^{2}\int \phi(|\nabla v_i|)dx +$$

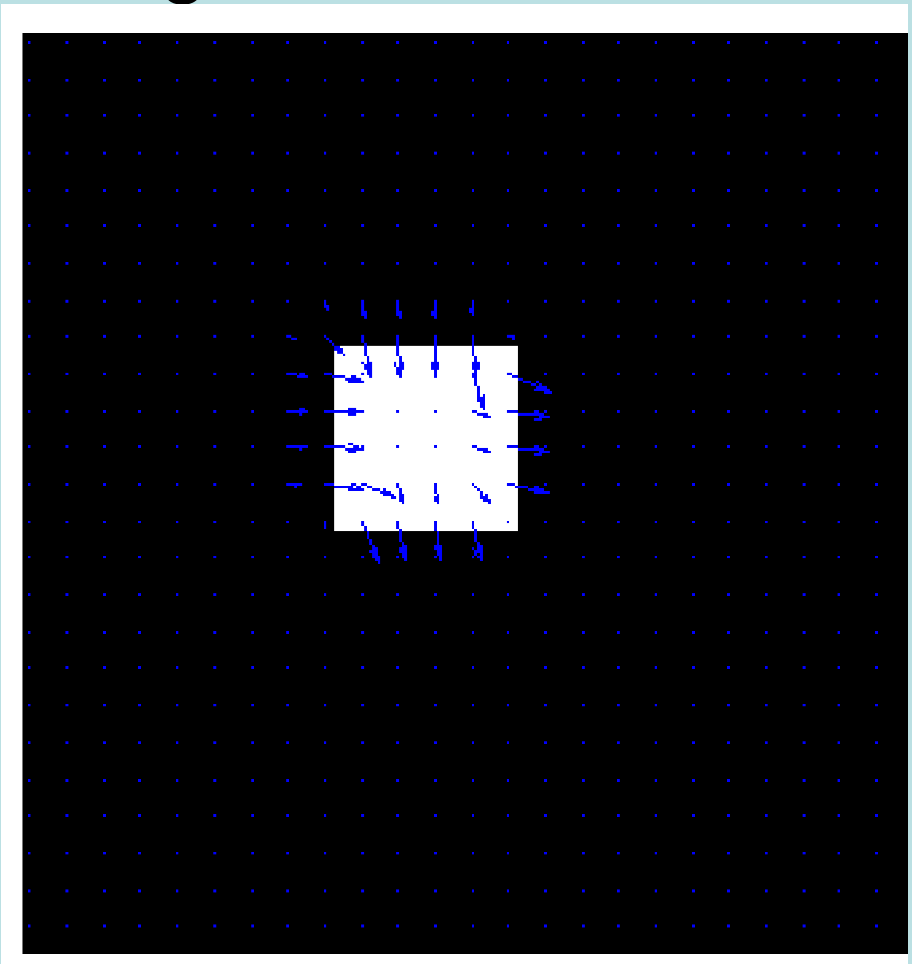
$$+\gamma \int_{\Omega} c(|\nabla I|)|\mathbf{v}|^2 dx$$

Homogenous term

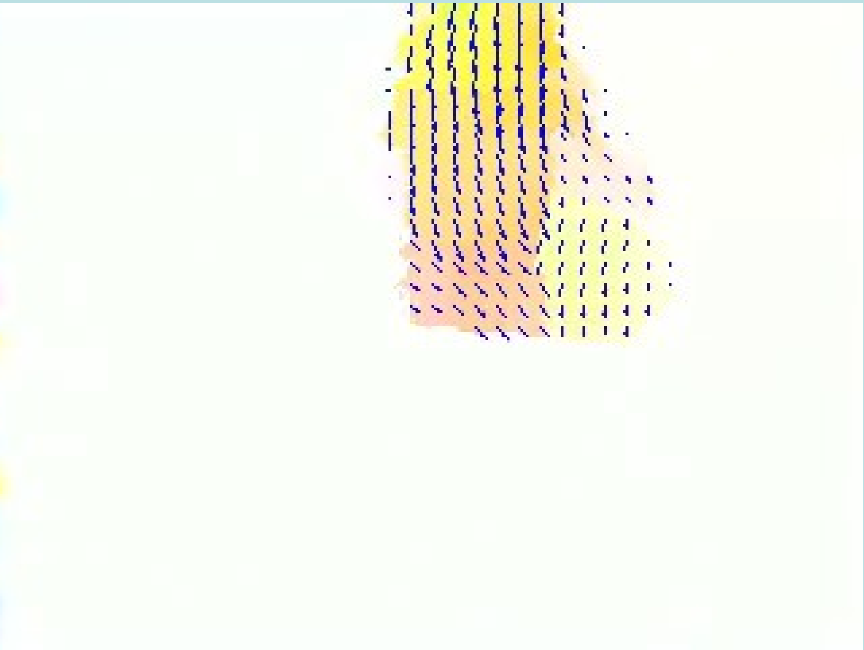
$$\lim_{s \to 0} c(s) = 1 \qquad \lim_{s \to +\infty} c(s) = 0$$

Example

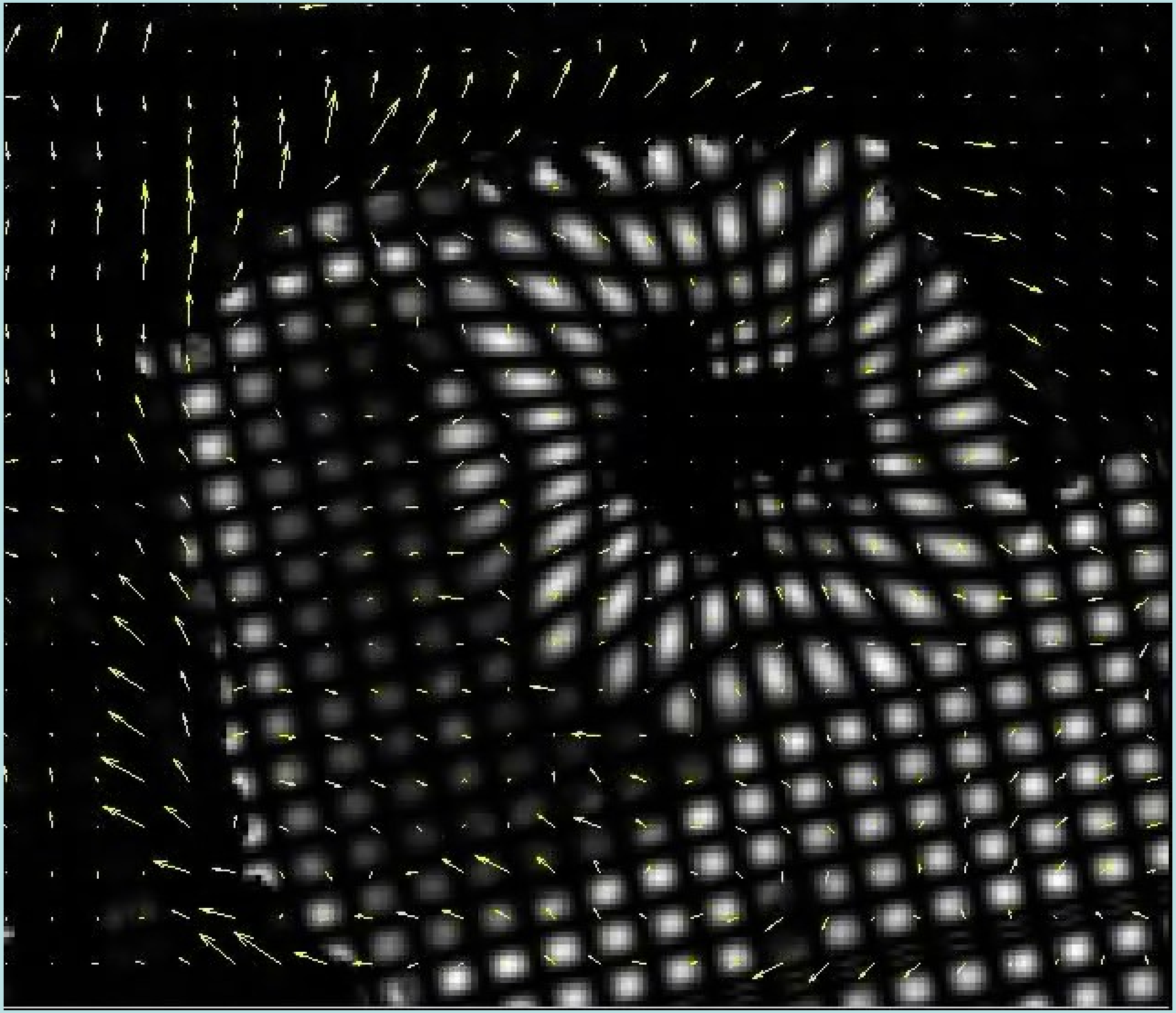
- With the homogeneous term



Security Camera



Cardiac MR example



Hierarchical OF