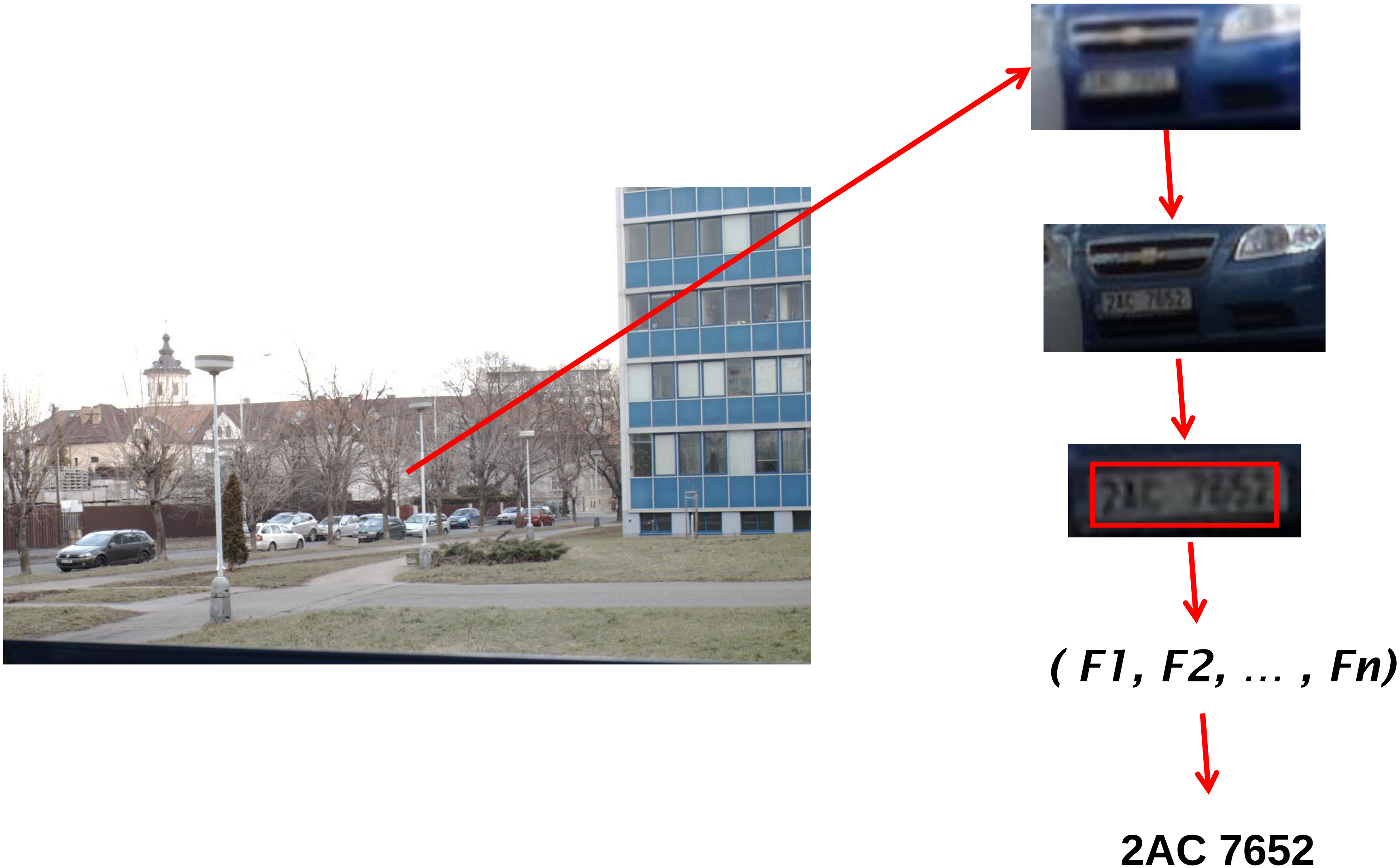
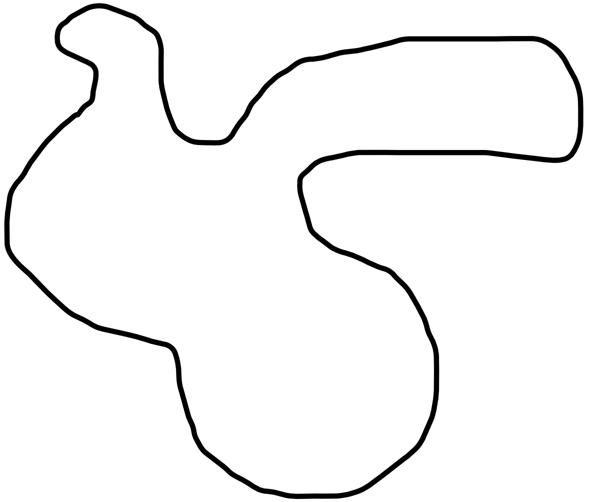

(F1, F2, ... , Fn)
2AC 7652

What is a 2-D object?

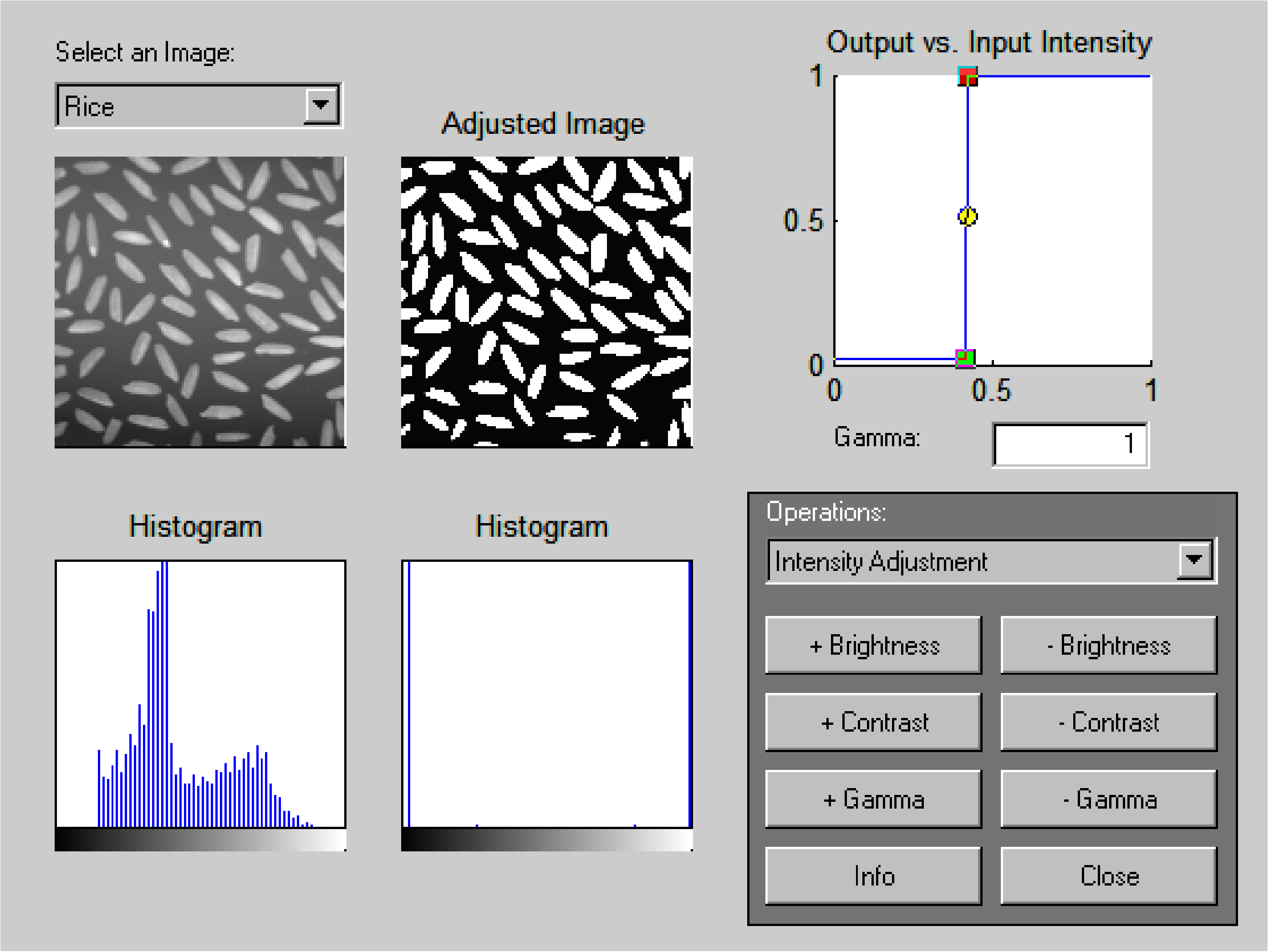
Binary

Finite

Boundary – a simple closed curve or a finite set of them



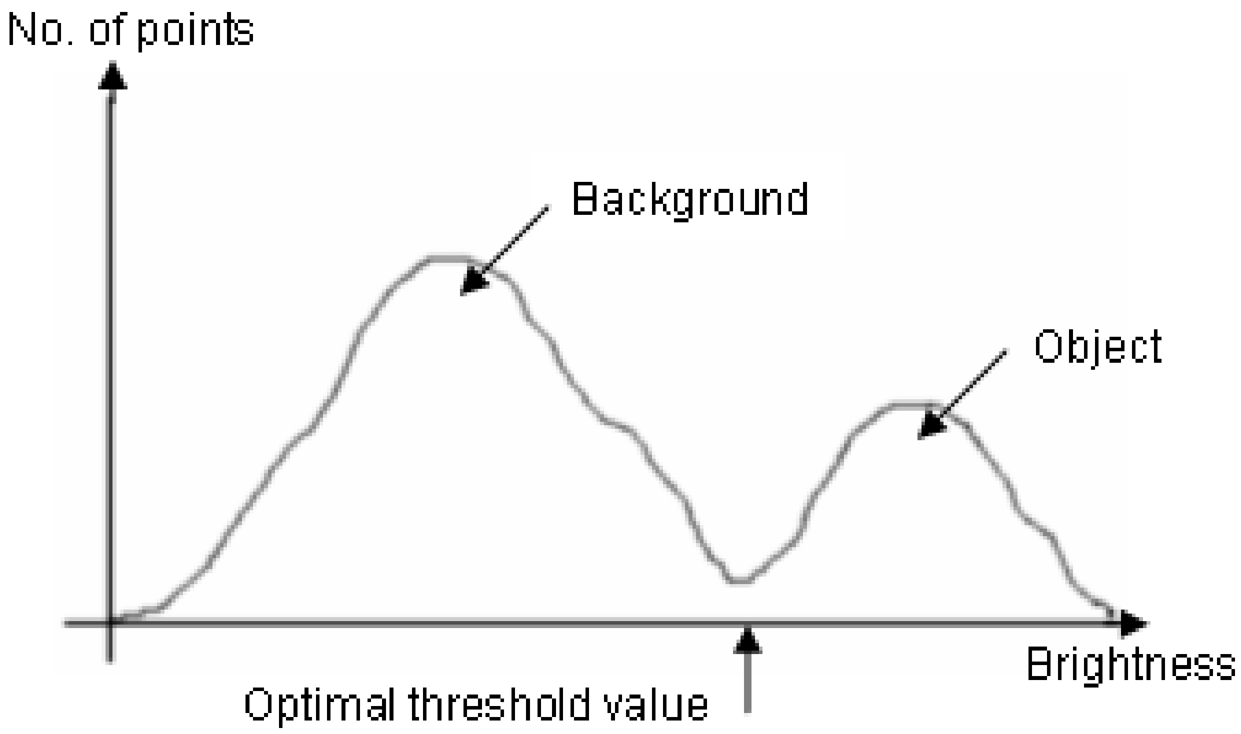
Thresholding



The Otsu's threshold

$$\sigma_w^2(t) = \omega_1(t)\sigma_1^2(t) + \omega_2(t)\sigma_2^2(t)$$

$$\sigma_b^2(t) = \sigma^2 - \sigma_w^2(t) = \omega_1(t)\omega_2(t)\left[\mu_1(t) - \mu_2(t)\right]^2$$



Desirable properties of the features

Invariance

Discriminability

Robustness

Efficiency, independence, completeness

Major categories of invariants

Simple "visual" shape descriptors

- compactness, convexity, elongation, ...

Transform coefficient invariants

- Fourier descriptors, wavelet features, ...

Point set invariants

- positions of dominant points

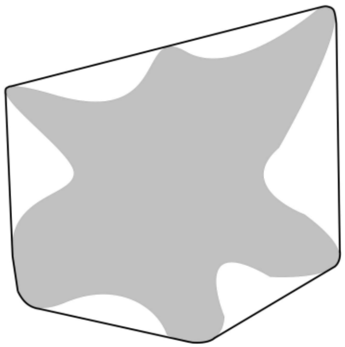
Differential invariants

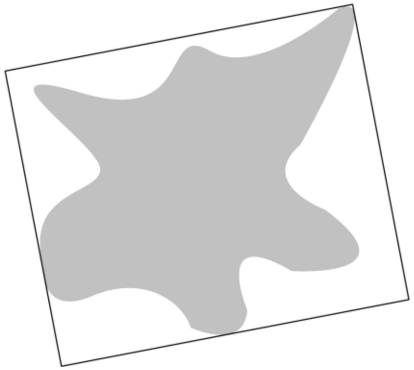
- derivatives of the boundary

Moment invariants

Visual features for binary objects

Simple features





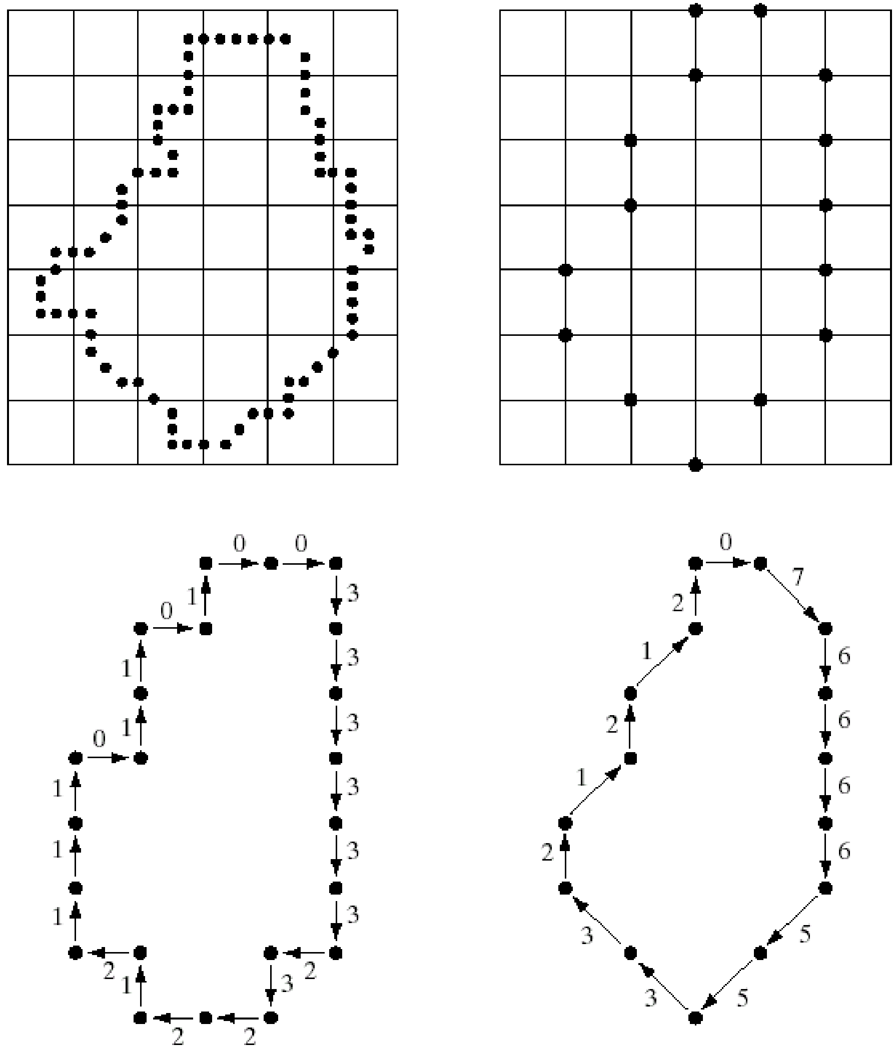
- Compactness $\frac{4\pi P}{O^2}$
- Convexity $\frac{P(A)}{P(C_A)}$
- Elongation
- Rectangularity
- Euler number

Visual features for binary objects

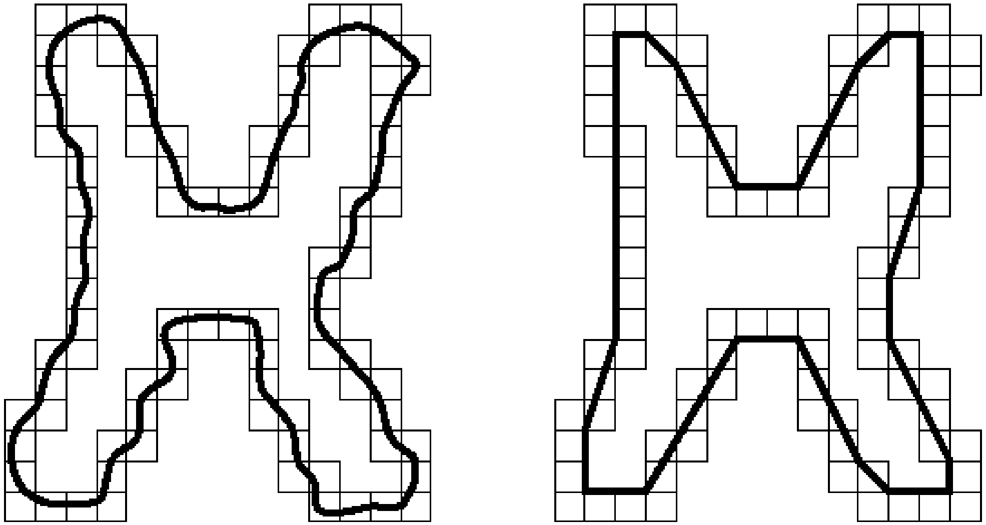
“Complete” features

- Chain code
- Polygonal approximation
- Shape vector
- Shape matrix
- Other encodings of the radial function

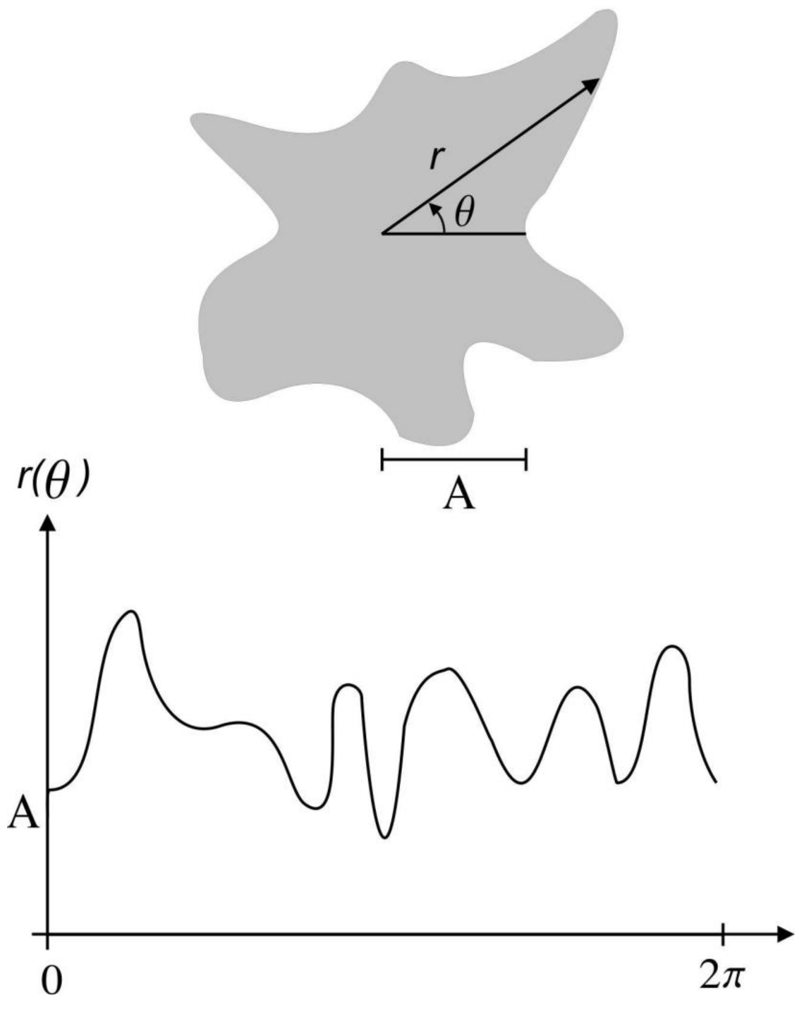
Chain code



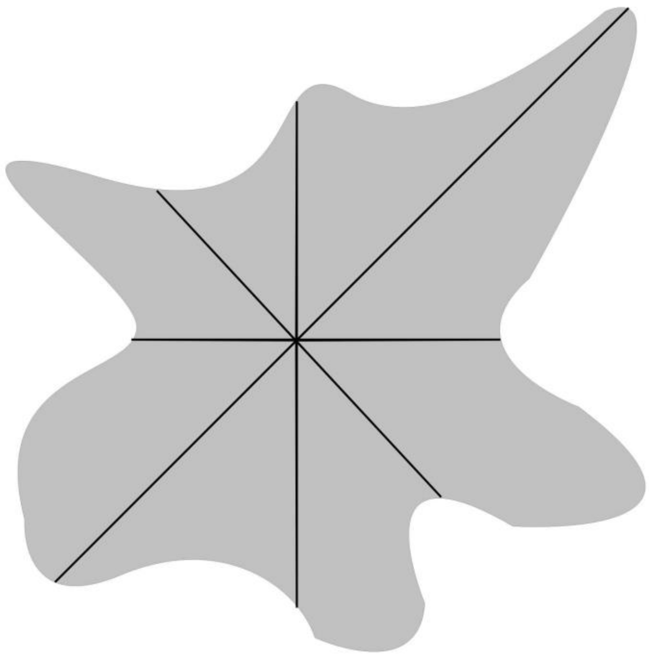
Polygonal approximation



Radial function



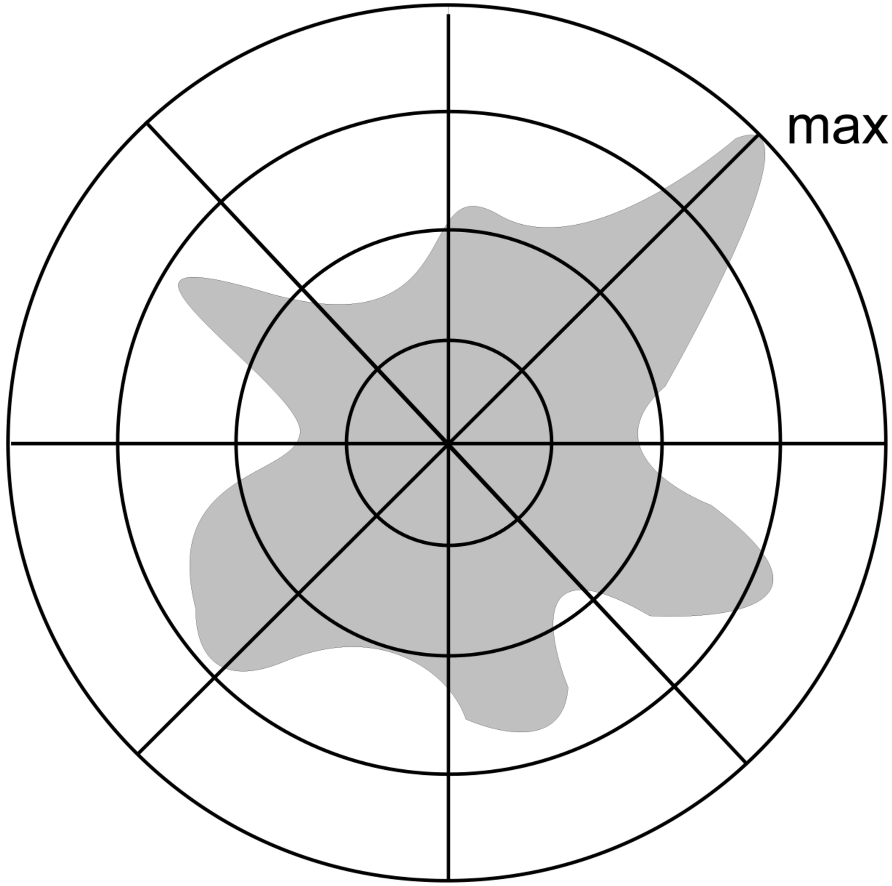
Shape vector



***v* = (*d1*, *d2*, ..., *dn*)**

n = 8

Shape matrix



$m = 4$

$n = 8$

$$B = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Moment invariants

Moments are "projections" of the image function into a polynomial basis

$f(x, y)$ – piecewise continuous image function defined on bounded $\Omega \subset \mathcal{R} \times \mathcal{R}$

$\{\mathcal{P}_{pq}(x, y)\}$ – set of polynomials defined on Ω

$$M_{pq} = \int\int \mathcal{P}_{pq}(x, y) f(x, y) dx dy$$

Common types of moments

Geometric moments

$$m_{pq}^{(f)} = \int_{R^2} x^p y^q f(x,y) dx dy$$

Geometric moments – the meaning

$$m_{pq}^{(f)} = \int_{R^2} x^p y^q f(x,y)\,dx\,dy$$

0th order - area

1st order - center of gravity

$$x_t = \frac{m_{10}}{m_{00}}, \qquad y_t = \frac{m_{01}}{m_{00}}$$

2nd order - moments of inertia

3rd order - skewness

Invariants to translation

Central moments

$$\mu_{pq}^{(f)} = \int_{R^2} (x - x_t)^p (y - y_t)^q f(x, y) dx dy$$

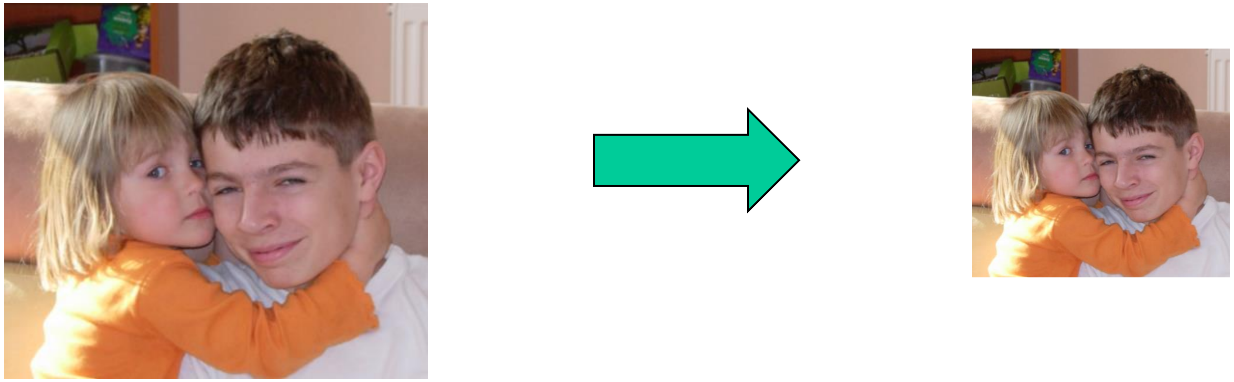
$$x_t = m_{10}/m_{00}, \quad y_t = m_{01}/m_{00}$$

$$\mu_{pq} = \sum_{k=0}^{p} \sum_{j=0}^{q} \binom{p}{k} \binom{q}{j} (-1)^{k+j} x_t^k y_t^j m_{p-k,q-j}$$

Invariants to translation and scaling

Normalized central moments

$$\nu_{pq} = \frac{\mu_{pq}}{\mu_{00}^{w}} \qquad w = \frac{p+q}{2} + 1$$



Invariants to translation and scaling

Normalized central moments

$$\nu_{pq} = \frac{\mu_{pq}}{\mu_{00}^{w}} \qquad w = \frac{p+q}{2} + 1$$

Invariants to rotation

M.K. Hu, 1962 - 7 invariants of the 3rd order

$$\phi_1 = \mu_{20} + \mu_{02}$$

$$\phi_2 = (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2$$

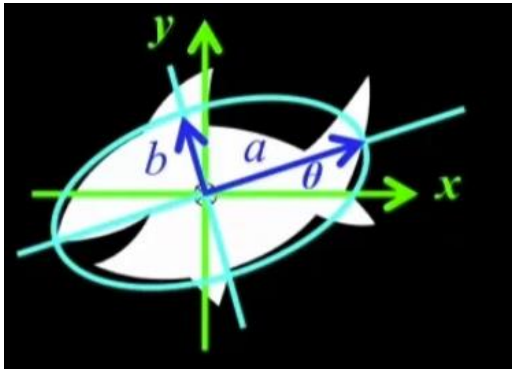
$$\phi_3 = (\mu_{30} - 3\mu_{12})^2 + (3\mu_{21} - \mu_{03})^2$$

$$\phi_4 = (\mu_{30} + \mu_{12})^2 + (\mu_{21} + \mu_{03})^2$$

$$
\begin{aligned}
\phi_5 &= (\mu_{30} - 3\mu_{12})(\mu_{30} + \mu_{12})((\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2) \\
&+ (3\mu_{21} - \mu_{03})(\mu_{21} + \mu_{03})(3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2)
\end{aligned}
$$

$$
\begin{aligned}
\phi_6 &= (\mu_{20} - \mu_{02})((\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2) + \\
&\quad 4\mu_{11}(\mu_{30} + \mu_{12})(\mu_{21} + \mu_{03})
\end{aligned}
$$

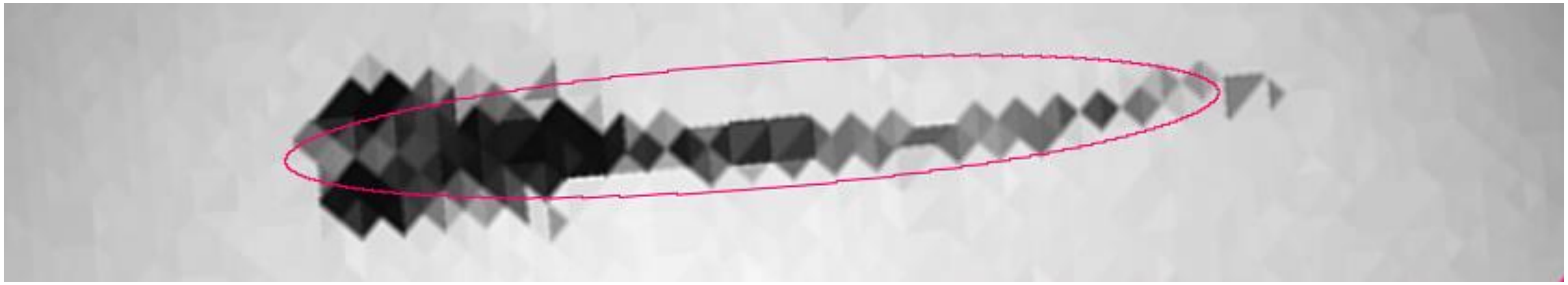
$$
\begin{aligned}
\phi_7 &= (3\mu_{21} - \mu_{03})(\mu_{30} + \mu_{12})((\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2) \\
&- (\mu_{30} - 3\mu_{12})(\mu_{21} + \mu_{03})(3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2)
\end{aligned}
$$



Major axis orientation

$$\theta = \frac{1}{2}.tan^{-1}\left(\frac{2\mu'_{1,1}}{\mu'_{2,0} - \mu'_{0,2}}\right)$$

$$\mu'_{20} - \mu'_{02} \neq 0$$



LBP – local binary patterns

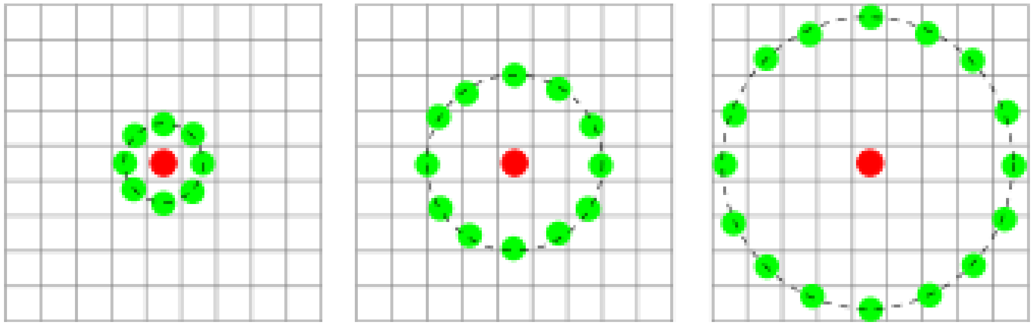
Popis textury

(1) – 16x16 buňky

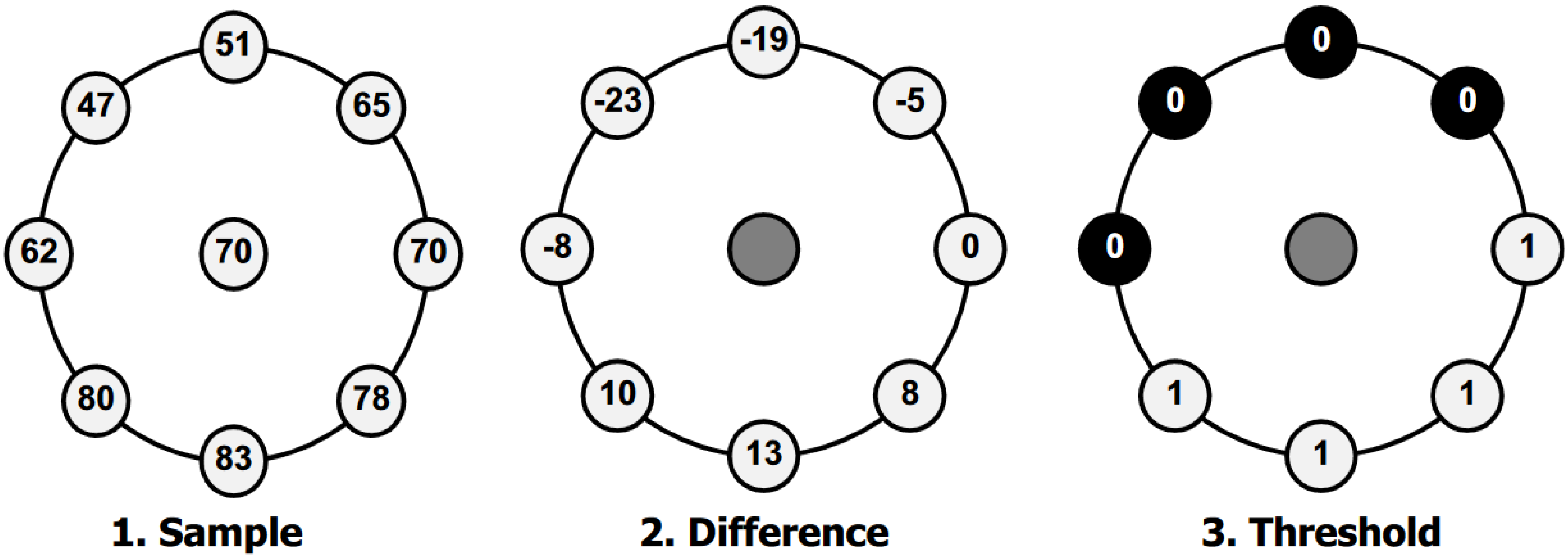
(2) – porovnej střed k 8 pixlum na obvodu

(3) – 0 když střed je menší, jinak 1

(4) - histogram



$$LBP_{P,R} = \sum_{p=0}^{P-1} s(g_p - g_c)2^p \qquad s(x) = \begin{cases} 1, & if\ x \geq 0; \\ 0, & otherwise. \end{cases}$$



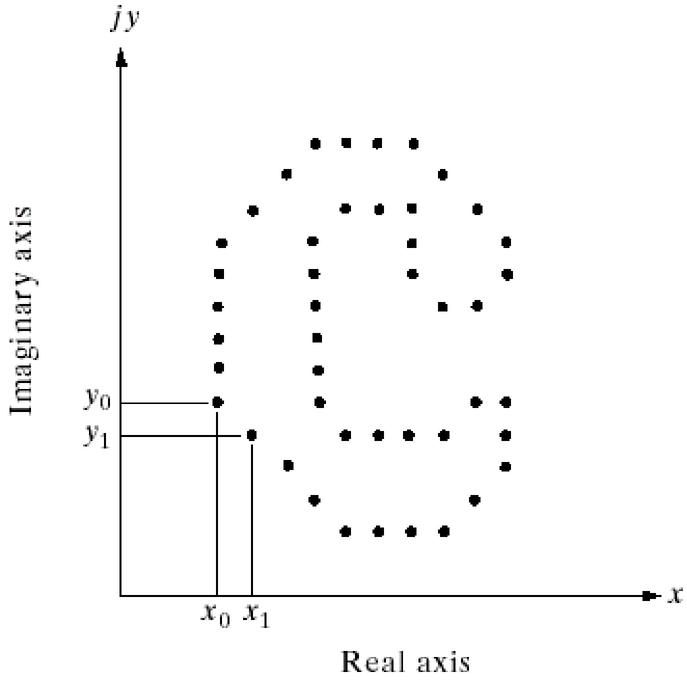
1*1 + 1*2 + 1*4 + 1*8 + 0*16 + 0*32 + 0*64 + 0*128 = 15

Transform coefficient features

- Fourier descriptors
- Wavelet-based features
- Other transform coefficients

Fourier descriptors

$$z_n = x_n + i \cdot y_n$$

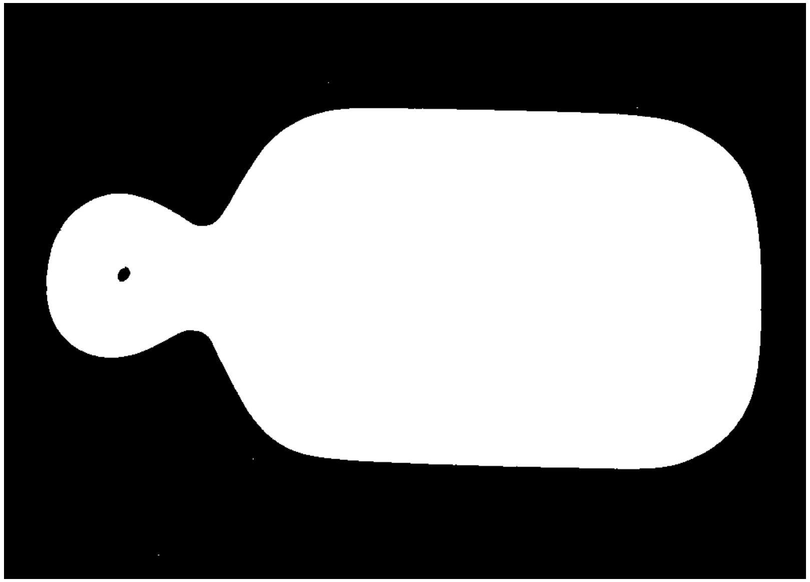


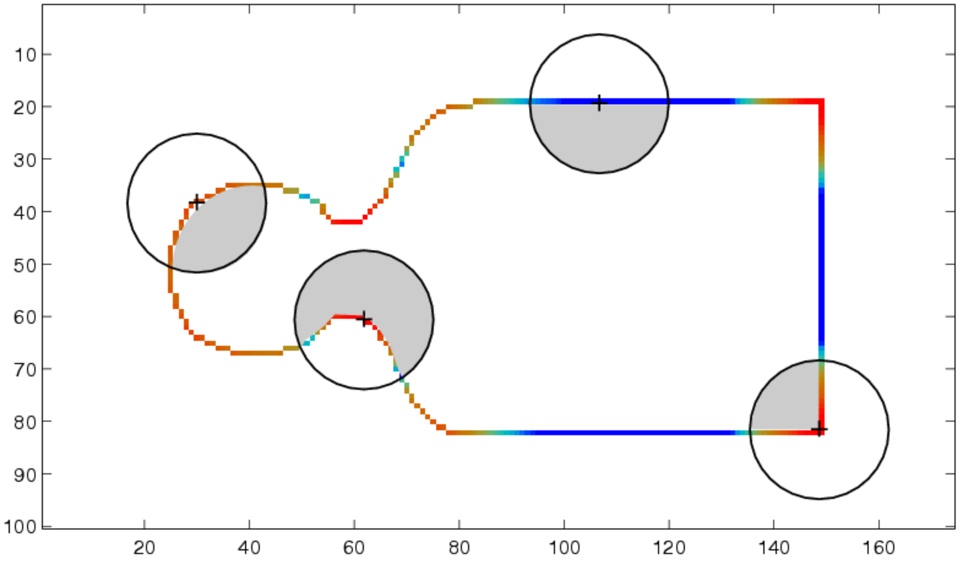
$$Z_k = \sum_{n=0}^{N-1} z_n e^{-2\pi ikn/N}$$

$$C_k = |Z_k|/|Z_1|, \ k = 2, 3, \ldots$$

Differential invariants – an example

$$c(t) = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$$





Affine-invariant radial vectors

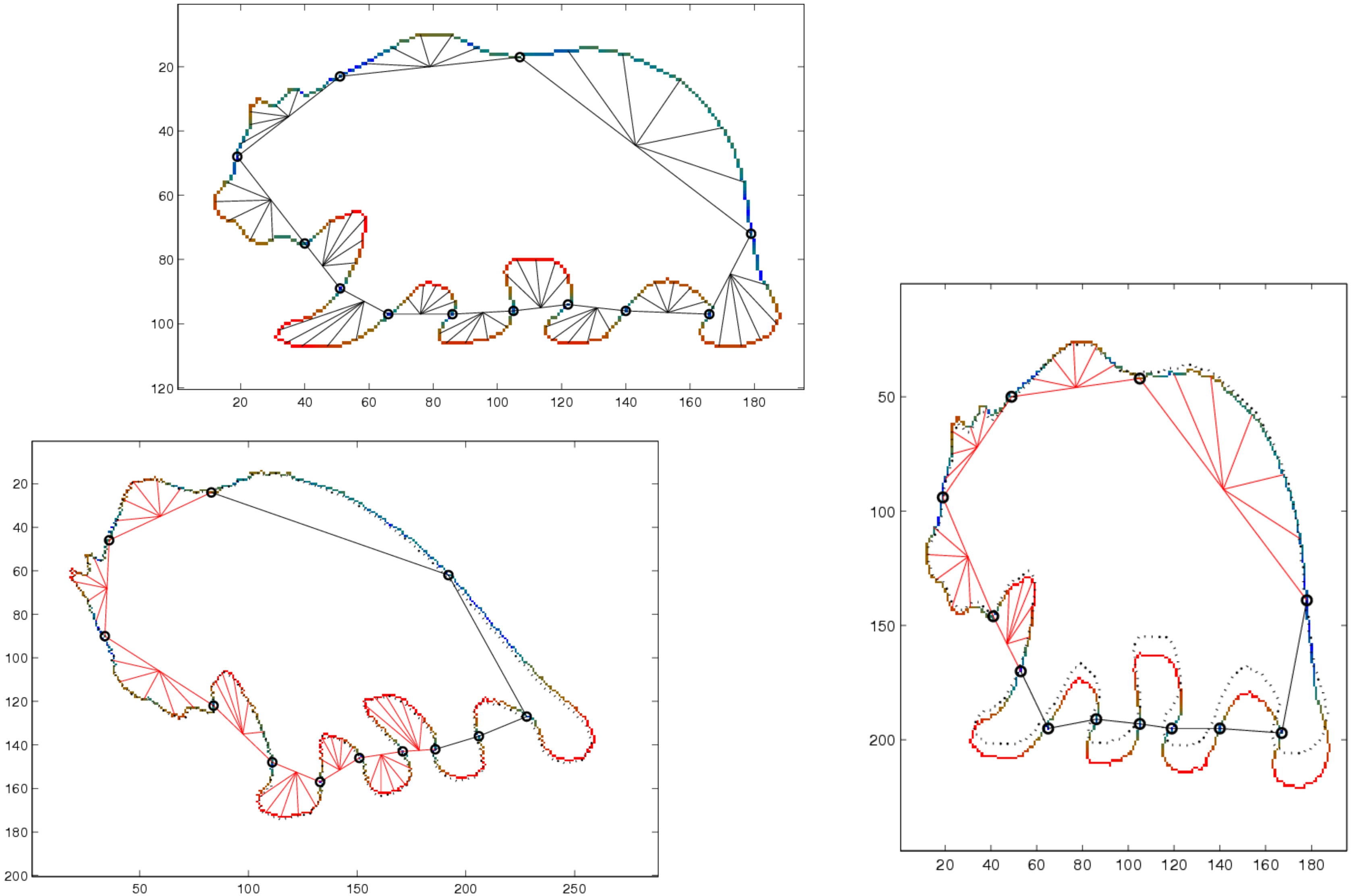


Image acquisition model

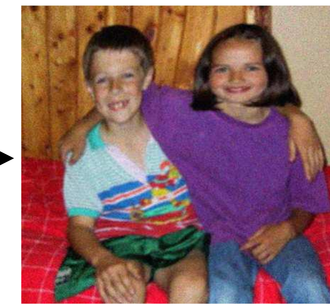
original scene

degradations

$$\mathcal{D} = \mathcal{Q}\mathcal{S}\mathcal{D}_{\mathcal{I}}\mathcal{D}_{\mathcal{G}}$$

noise

+



acquired image

$$\mathcal{D}_{\mathcal{G}}(f)(x,y) = f(\tau(x,y))$$

$$\mathcal{D}_{\mathcal{I}}(f)(x,y) = \int_{\Lambda}\int_{T}\int\int h(x,y,a,b,\lambda,t)f(a,b)\,da\,db\,d\lambda\,dt$$

Simplified space-invariant blur model

Flat scene

Constant motion

$$z(x) = (h * u)(x) + n(x)$$

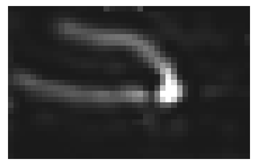
h(x) is the PSF of the camera

Understanding PSF

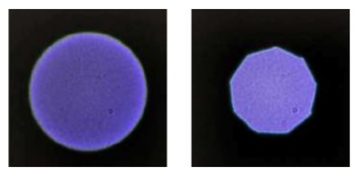
PSF is the response to an ideal point source

$$z(x) = (h * u)(x) + n(x)$$

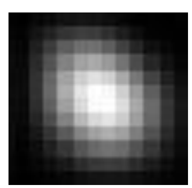
- *camera shake/motion*

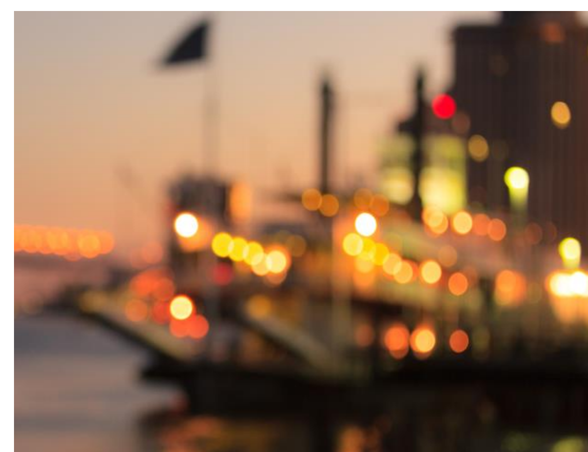


- *out-of-focus*



- *turbulence*





Noisy case

$$g = \mathcal{D}(f) + n$$

Noise makes the problem ill-conditioned and difficult to handle

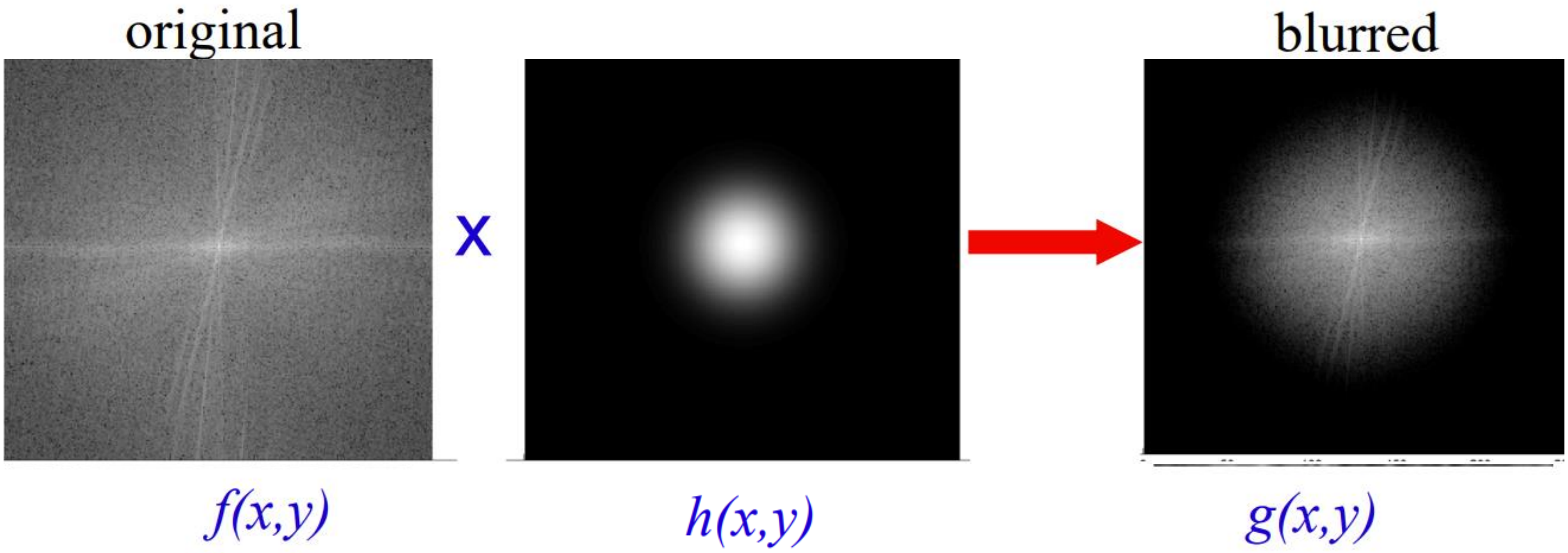
$$\hat{f} \neq \mathcal{D}^{-1}(g)$$

Denoising and/or regularization techniques are required.

Image restoration categories

- **From a single image (single-channel)**
  - PSF is completely known
  - PSF is of a known parametric shape
  - PSF is constant and unknown
  - PSF is variable and unknown
- **From multiple images (multi-channel)**

PSF completely known



i.e. : $g(x,y) = h(x,y) * f(x,y)$

$h(x,y)$ is the impulse response or point spread function of the imaging system

A. Zisserman

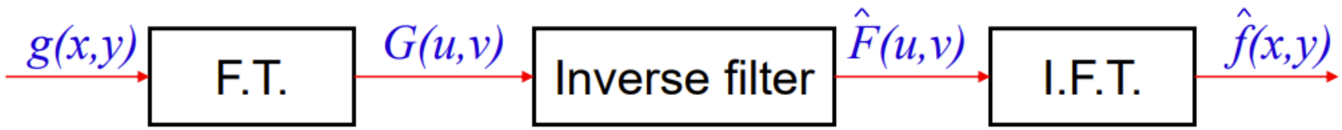
Intuitive solution to the inverse problem

No noise, PSF known – Fourier transform

$$g(x,y) = h(x,y) * f(x,y)$$

$$G(u,v) = H(u,v)\, F(u,v)$$

$$\hat{F}(u,v) = G(u,v) \,/\, H(u,v)$$

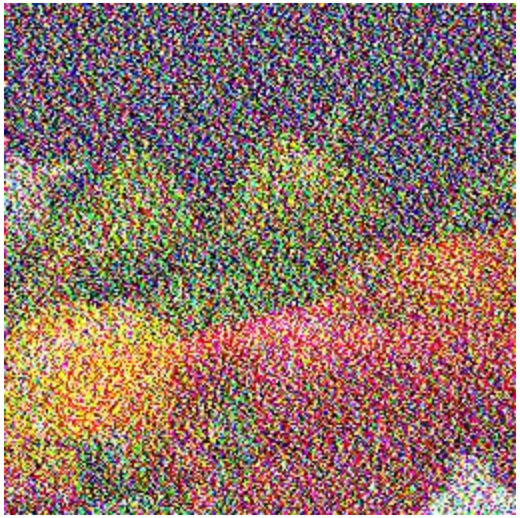


Intuitive solution to the inverse problem

... does not work on images with noise

$$G = F \cdot H + N$$

$$F = \frac{G}{H} - \frac{N}{H}$$

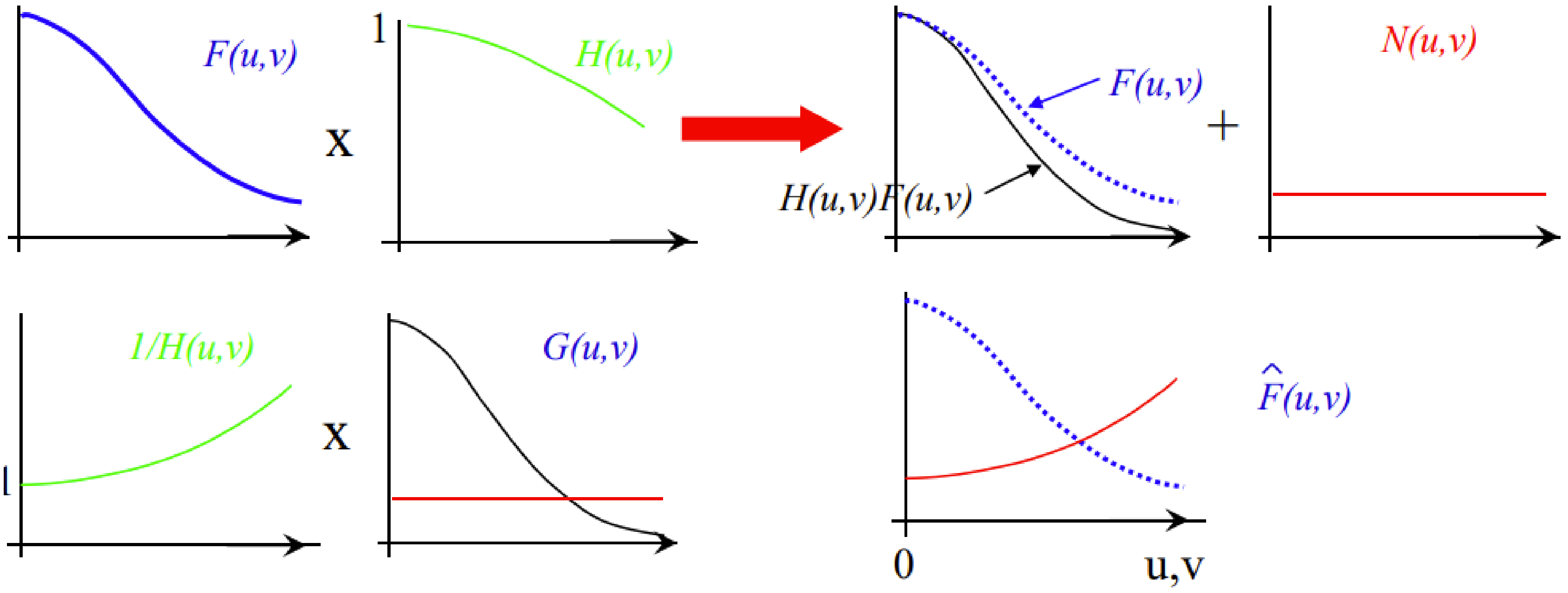


Intuitive solution to the inverse problem

... does not work on images with noise

$$G(u,v) = H(u,v)\, F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = G(u,v) \,/\, H(u,v) = F(u,v) + N(u,v) \,/\, H(u,v)$$



Wiener filter

restoration with minimum mean-square error (MSE)

$$\min_W \; e^2 = E\{(f - \widehat{f})^2\}$$

restrict to linear space-invariant filter

$$\widehat{f}(x, y) = w(x, y) * g(x, y)$$

find "optimal" linear filter W(u,v) with min. MSE

Wiener filter

$$W(u,v) = \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}$$

- $S_n(u,v)/S_f(u,v) \approx SNR^{-1}$

$S_f(u,v) = |F(u,v)|^2$ power spectral density of $f(x,y)$

$S_\eta(u,v) = |N(u,v)|^2$ power spectral density of $\eta(x,y)$

Wiener filter

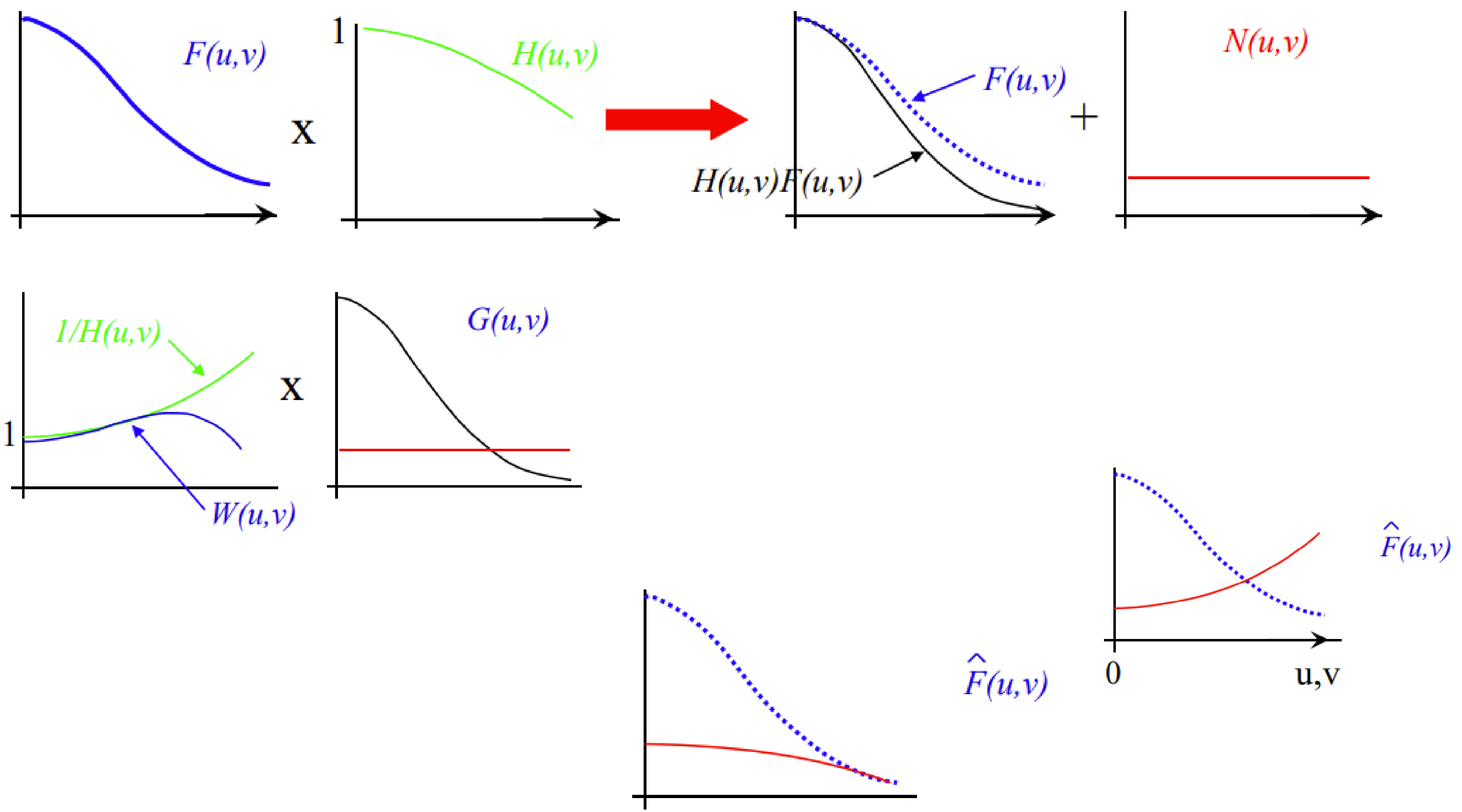
$$W(u,v) = \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}$$

$$K(u,v) = S_\eta(u,v)/S_f(u,v)$$

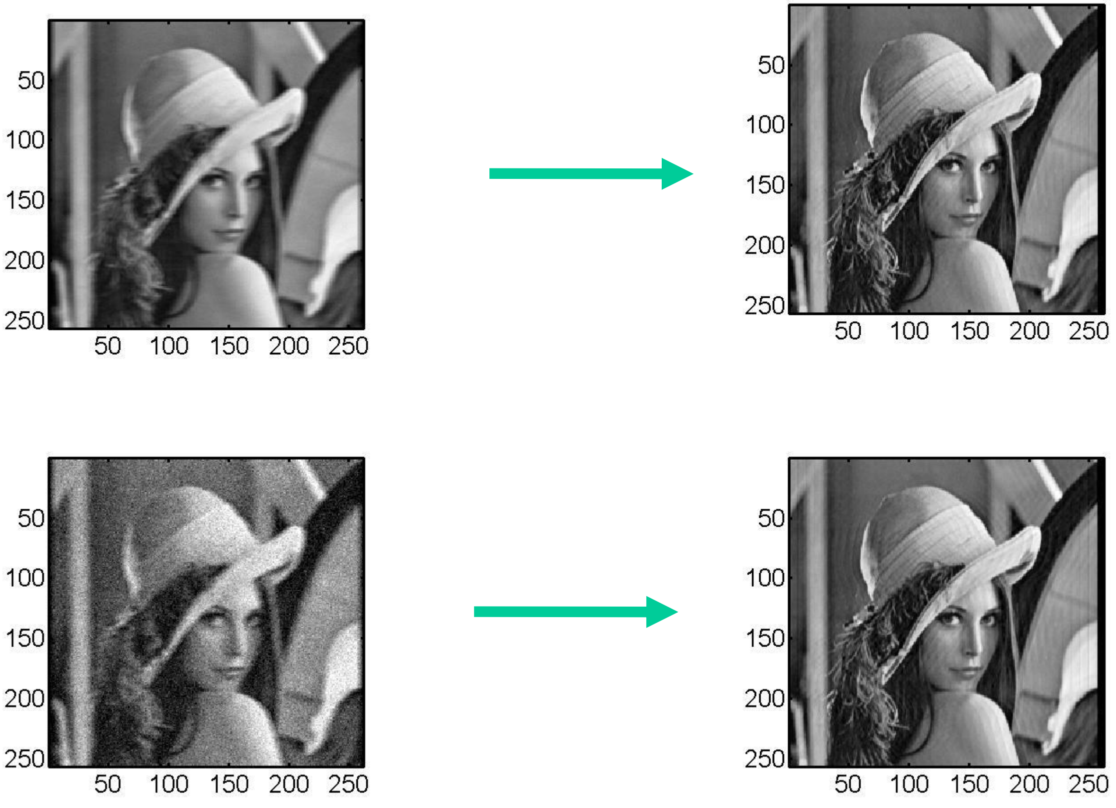
- If $K = 0$ then $W(u,v) = 1 / H(u,v)$, i.e. an inverse filter
- If K >> $|H(u,v)|$ *for large* u,v, then high frequencies are attenuated

- $|F(u,v)|$ and $|N(u,v)|$ are often known approximately, or
- K is set to a constant scalar which is determined empirically

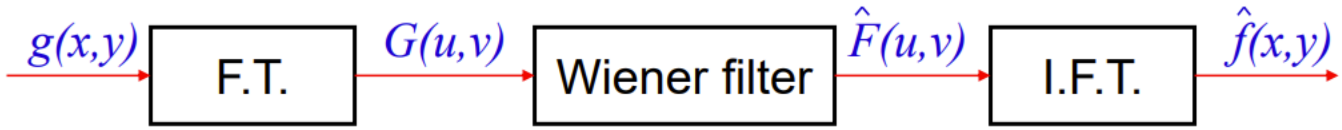
Wiener filter



Wiener filter



Wiener filter



blur σ = 1.5 pixels

noise σ = 0.3 grey levels

$$\hat{F}(u,v) = W(u,v)\, G(u,v)$$

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K(u,v)}$$

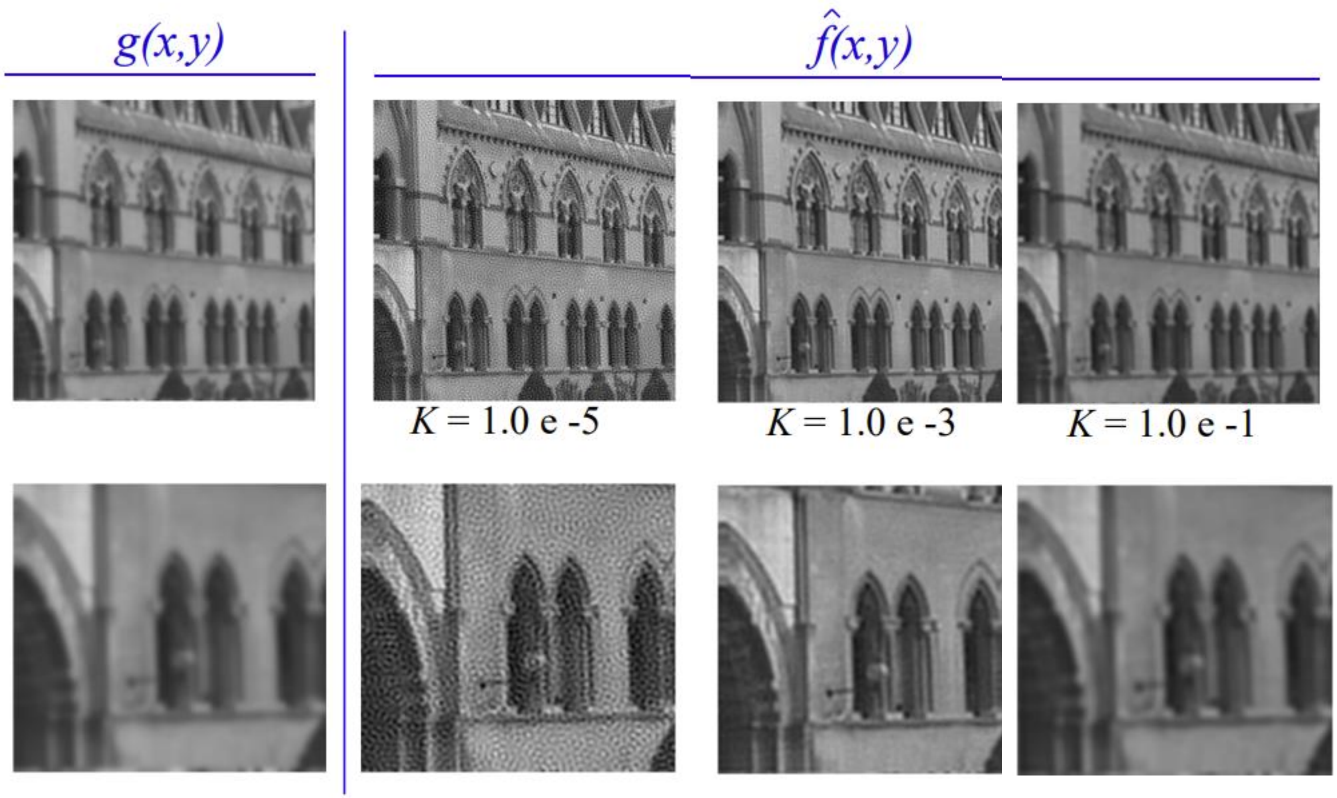
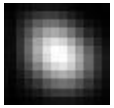


Image restoration flowchart

Blurred image



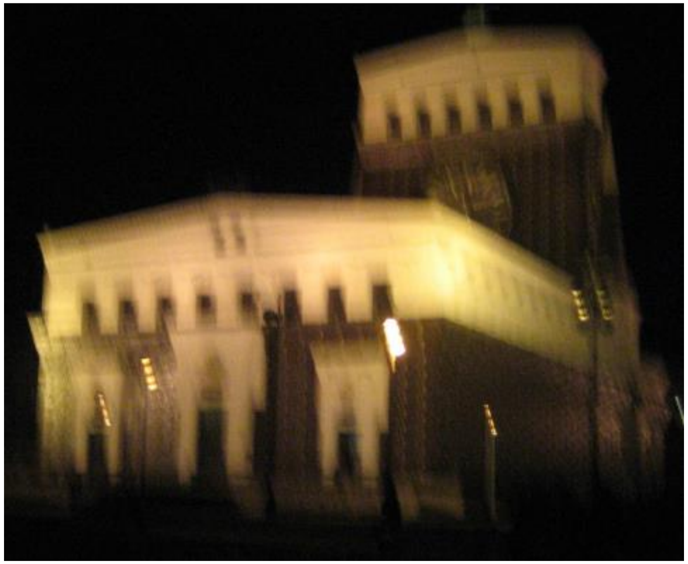
PSF estimation

Deconvolution

$$h * u = \int u(x-s, y-t)h(s,t)\,dsdt$$

Restored image

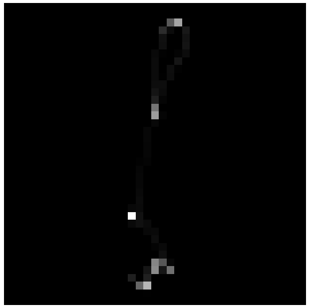
PSF estimation in smartphones



- Camera shake blur
- Dominant motion - rotation

PSF estimation in smartphones

- Using accelerometers and/or gyroscopes
- Rotation and translation of the phone

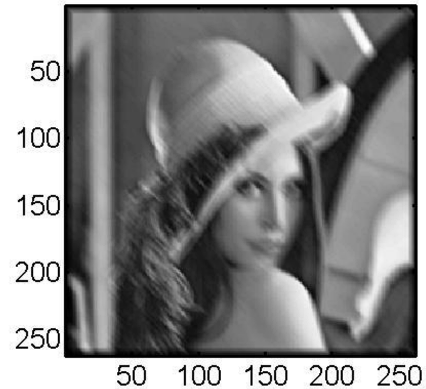


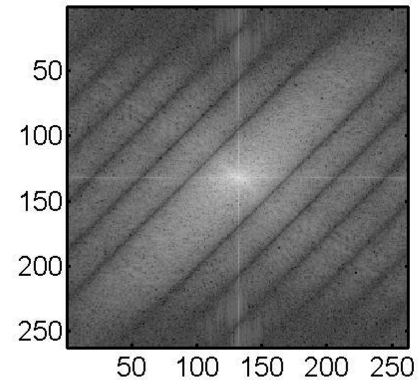
Restoration categories

- **PSF is completely known**
- **PSF is constant and of a known parametric shape**
- **PSF is constant and unknown**
- **PSF is variable and unknown**

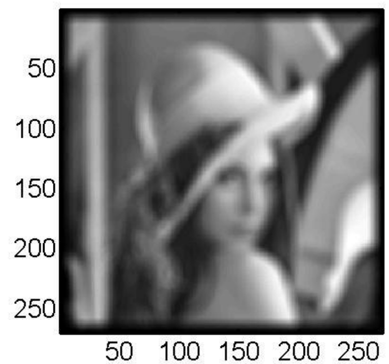
Spectrum of a degraded image

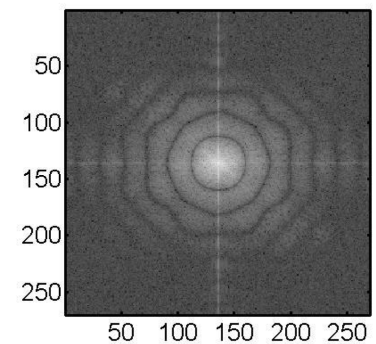
Motion blur



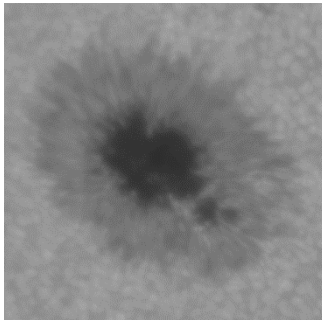


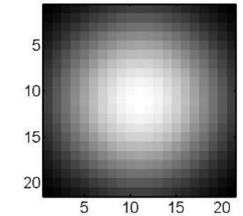
Out-of-focus blur





Atmospheric turbulence blur

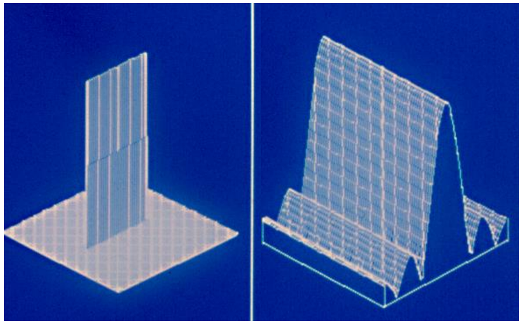




Common point-spread functions

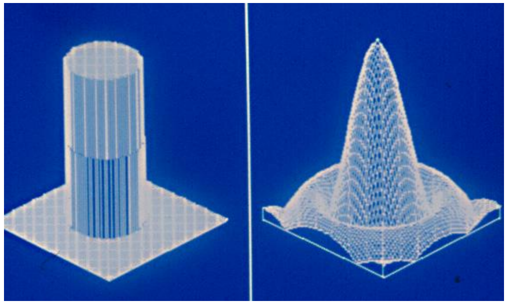
Motion blur:

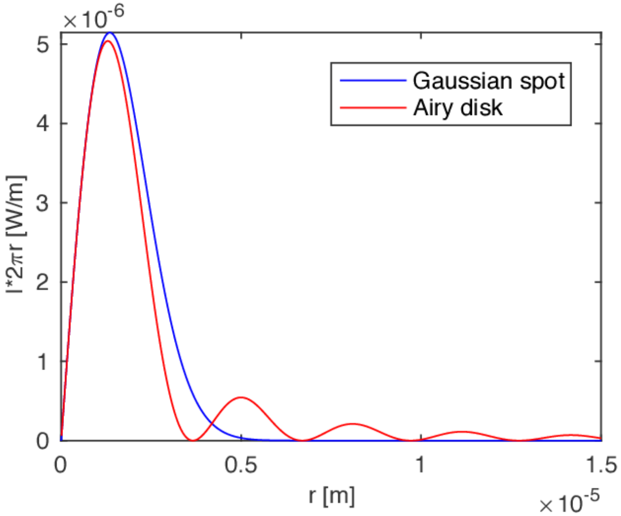
1-D rectangular pulse, FT = sinc(u)

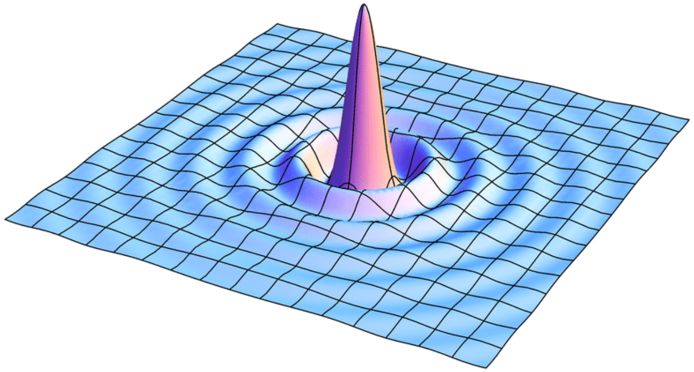


Out-of-focus blur:

Cylinder, FT = B(r)/r



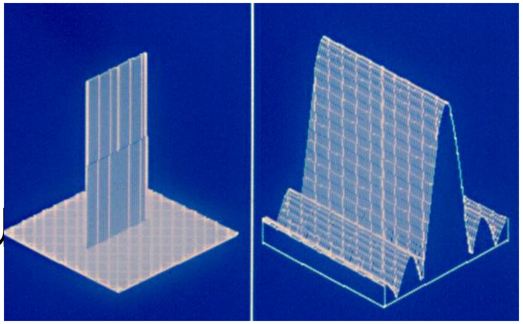




Common point-spread functions

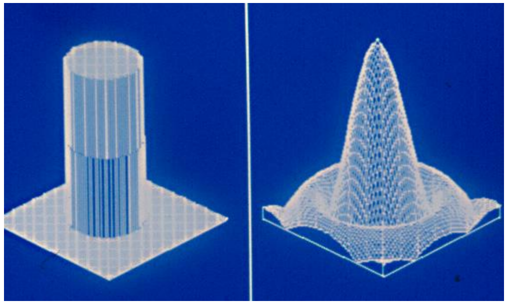
Motion blur:

1-D rectangular pulse, FT = sinc(u



Out-of-focus blur:

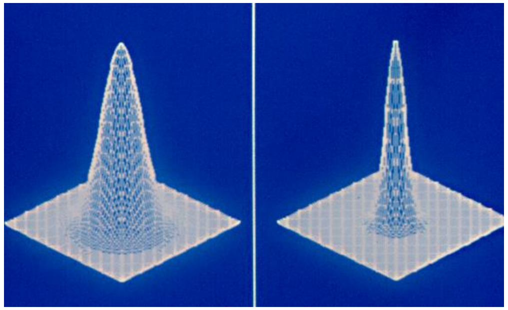
Cylinder, FT = B(r)/r



Atmospheric turbulence:

Gaussian G(d),

FT = Gaussian G(1/d)



Restoration categories

- **PSF is completely known**
- **PSF is constant and of a known parametric shape**
- **PSF is constant and unknown**
- **PSF is variable and unknown**

Blind deconvolution

$$z(x) = (h * u)(x) + n(x)$$

- almost impossible to resolve
- solution ambiguity

$$z(x) = ((h_1 * h_2 * \cdots * h_L) * u)(x) + n(x)$$

Alternating Minimization

$$\min_{u,h} E(u,h) = \min_{u,h} \frac{1}{2}\|h * u - z\|^2 + \lambda Q(u) + \gamma R(h)$$

- Alternating Minimization

  1. *u-step:* $\tilde{u} = \arg\min_u E(u, \tilde{h})$

  2. *h-step:* $\tilde{h} = \arg\min_h E(\tilde{u}, h)$

  3. *repeat 1 and 2.*

Restoration categories

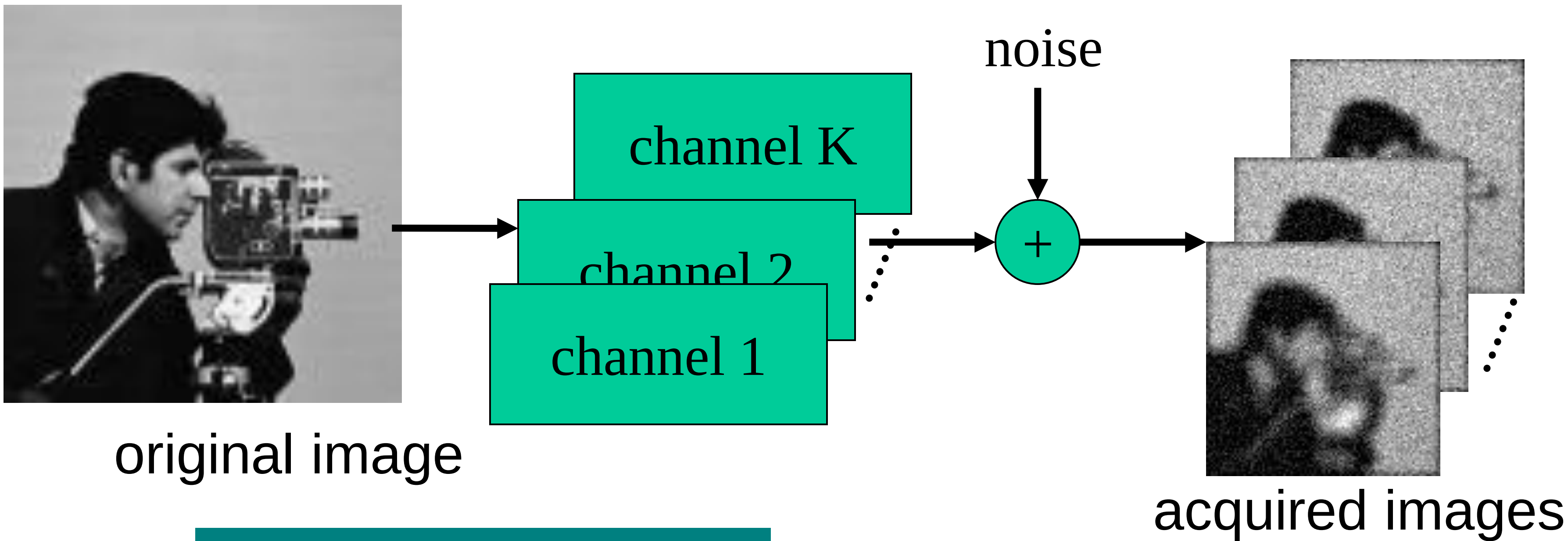
- **PSF is completely known**
- **PSF is constant and of a known parametric shape**
- **PSF is constant and unknown**
- **PSF is variable and unknown**

Multichannel image restoration

Assumptions:

- Several input images of the same scene are available
- They are blurred by convolution with different convolution kernels
- The original scene does not change during the acquisitions

Multichannel acquisition model



$$[u * h_k](x, y) + n_k(x, y) = z_k(x, y)$$

MC Blind Deconvolution

- System of integral equations

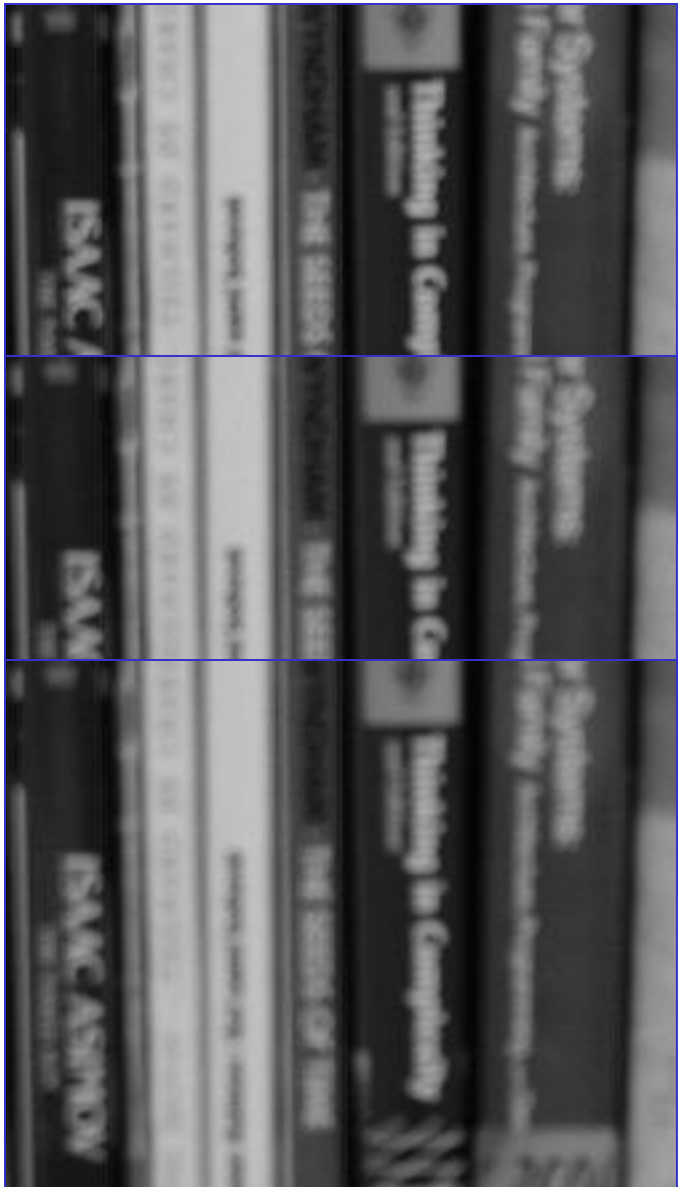
  (ill-posed, underdetermined)

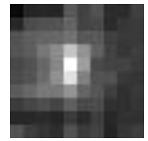
$$z_k(x) = (h_k * u)(x) + n_k(x)$$

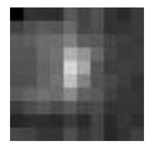
Energy minimization problem (well-posed)

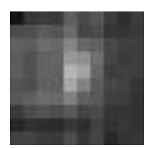
$$E(u, \{h_i\}) = \frac{1}{2}\sum_{i=1}^{K} \|h_i * u - z_i\|^2 + \lambda Q(u) + \gamma R(\{h_i\})$$

Out-of-focus Camera









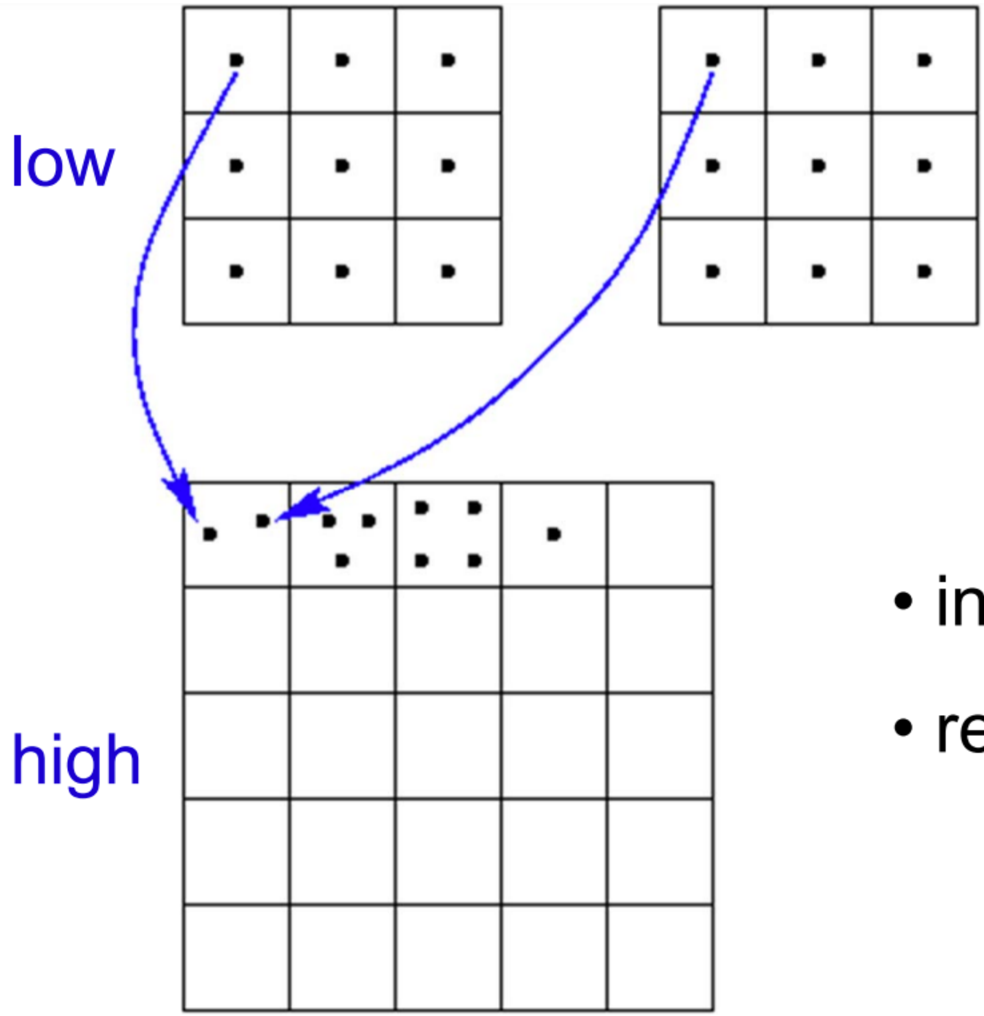
reconstructed

in focus

Aliasing

- The loss of the high frequencies (details) due to an insufficient resolution of the camera

Superresolution



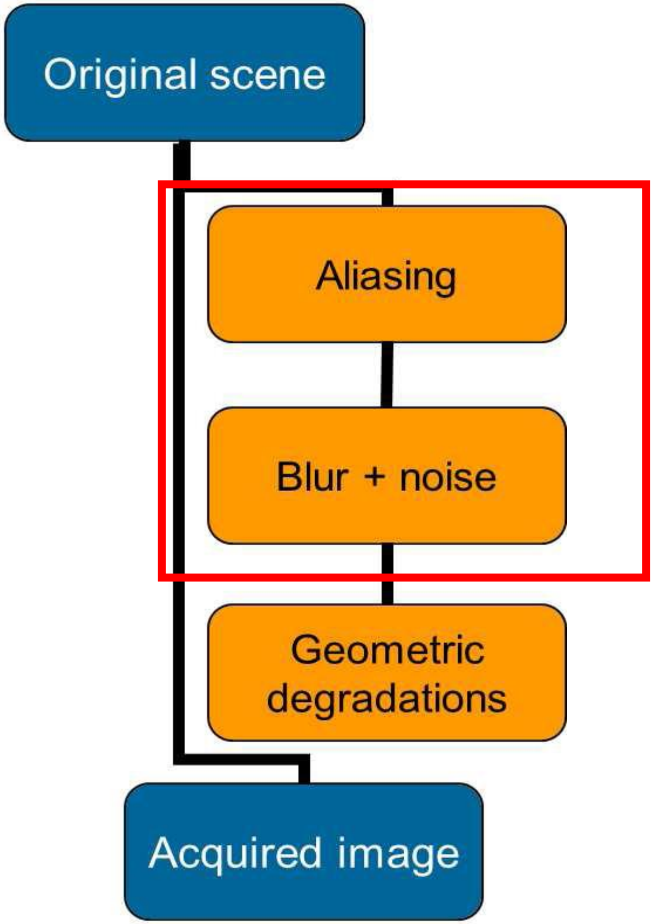
• • • more images

• increase resolution

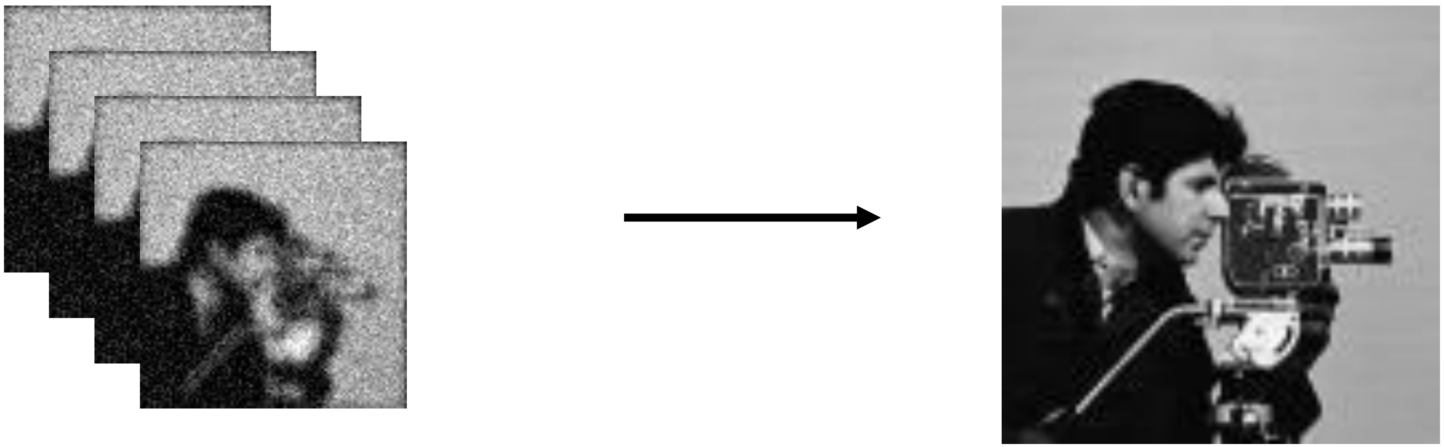
• reduce noise

Realistic superresolution

SR must include also de-blurring



Realistic superresolution



$$E(u,\{g_i\}) = \frac{1}{2}\sum_{i=1}^{K} \|D(g_i * u) - z_i\|^2 + \lambda Q(u) + \gamma R(\{g_i\})$$